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Lie-admissible invariant treatment of irreversibility for matter and antimatter at the classical and operator levels

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Summary. — It was generally believed throughout the 20-th century that irreversibility is a purely classical event without operator counterpart. However, a classical irreversible system cannot be consistently decomposed into a finite number of reversible quantum particles (and, vice versa), thus establishing that the origin of irreversibility is basically unknown at the dawn of the 21-st century. To resolve this problem, we adopt the historical analytic representation of irreversibility by Lagrange and Hamilton, that with external terms in their analytic equations; we show that, when properly written, the brackets of the time evolution characterize covering Lie-admissible algebras; we prove that the formalism has a fully consistent operator counterpart given by the Lie-admissible branch of hadronic mechanics; we identify mathematical and physical inconsistencies when irreversible formulations are treated with the conventional mathematics used for reversible systems; we show that, when the dynamical equations are treated with a novel irreversible mathematics, Lie-admissible formulations are fully consistent because invariant at both the classical and operator levels; and we complete our analysis with a number of explicit applications to irreversible processes in classical mechanics, particle physics and thermodynamics. The case of closed-isolated systems verifying conventional total conservation laws, yet possessing an irreversible structure, is treated via the simpler Lie-isotopic branch of hadronic mechanics. The analysis is conducted for both matter and antimatter at the classical and operator levels to prevent insidious inconsistencies occurring for the sole study of matter or, separately, antimatter.

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1. – Introduction

1·1. *The scientific imbalance caused by irreversibility.* – As is well known, rather vast studies have been conducted in the 20-th century in attempting a reconciliation of the manifest irreversibility in time of our macroscopic environment with the reversibility of

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quantum mechanics, resulting in the rather widespread belief that irreversibility solely exists at the macroscopic level because it "disappears" at quantum-mechanical level (see, *e.g.*, the reprint volume by Shoeber [1] and vast literature quoted therein).

The above belief created a scientific imbalance because irreversible systems were treated with the mathematical and physical formulations developed for reversible systems. Since these formulations are themselves reversible, the attempt in salvaging quantum mechanics *vis a vis* the irreversibility of physical reality caused serious limitations in virtually all quantitative sciences.

The imbalance originated from the fact that all used formulations were of *Hamiltonian type* (*i.e.* the formulations are entirely characterized by the sole knowledge of the Hamiltonian), under the awareness that all known Hamiltonians are reversible because all known potentials (such as the Coulomb potential $V(r) = q_1 q_2 / r$) are reversible.

The academic belief of the purely classical character of irreversibility was disproved in 1985 by Santilli [2] via the following theorem whose proof is instructive for serious scholars in the field

Theorem 1.1: *A classical irreversible system cannot be consistently reduced to a finite number of quantum particles all in reversible conditions and, vice versa, a finite ensemble of quantum particles all in reversible conditions cannot consistently reproduce an irreversible macroscopic system under the correspondence or other principles.*

The implications of the above theorem are rather deep because the theorem establishes that, rather than adapting to quantum mechanics all possible physical events in the universe, it is necessary to seek a *covering of quantum mechanics* permitting a consistent treatment of elementary particles in irreversible conditions, where the notion of covering is intended to be such that, when irreversibility is removed, quantum mechanics is recovered identically and uniquely.

In the final analysis, the orbit of an electron in an atomic structure is indeed reversible in time, with consequential validity of quantum mechanics. However, the idea that the same electron has an equally reversible orbit when moving in the core of a star is repugnant to reason as well as scientific rigor because, *e.g.*, said idea would imply that the electron has to orbit in the core of a star with a locally conserved angular momentum (from the basic rotational symmetry of all Hamiltonians), with consequential direct belief of the existence of the perpetual motion within physical media.

In short, contrary to a popular belief throughout the 20-th century, Theorem 1.1 establishes that *irreversibility originates at the most primitive levels of nature, that of elementary particles, and then propagates all the way to our macroscopic environment.*

In this paper, we complete studies in the origin of irreversibility initiated by the author during his graduate studies in physics at the University of Torino, Italy, in the late 1960s [7-9], and continued at Harvard University under DOE support inthe late l970s [6, 11, 12], which studies achieved mathematical and physical consistency only recently. The decades required by the completion of the research are an indication of the complexity of the problem.

In fact, it was relativity easy to identify since the early efforts irreversible generalizations of Heisenberg's equations [7-9]. However, these generalized equations subsequently resulted to lack *invariance,* here intended as the capability by quantum mechanics of predicting the same numerical values for the same conditions but under the time evolution of the theory.

In turn, the achievement of invariance predictably required the laborious prior effort of identifying a *new irreversible mathematics,* namely, a new mathematics specifically constructed for the invariant treatment of irreversible systems while being a covering of conventional mathematics in the above indicated sense. Following the achievement of the new mathematics, the resolution of the scientific imbalance of the 20-th century caused by irreversibility was direct and immediate.

By looking in retrospective, rather than being demeaning for quantum mechanics, the search for its invariant irreversible covering has brought into full light the majestic axiomatic consistency of quantum mechanics, and how difficult resulted to be the preservation of the same axiomatic consistency at the covering irreversible level.

In other words, only art masterpieces, such as Michelangelo's *La Pietà,* are eternal. By comparison, quantitative sciences will never admit final theories because, no matter how majestic a given theory may appear at a given time, its surpassing by a broader theory for more complex physical conditions is only a matter of time.

As we shall see, the studies presented in this paper required several preceding contributions over an extended period of time, although their coordination for the invariant treatment of irreversibility is presented in this paper for the first time. A comprehensive presentation of these studies will appear in monographs [18c] following the publication of this paper.

1˙2. *Forgotten legacy of Newton, Lagrange and Hamilton.* – The scientific imbalance on irreversibility was created in the early part of the 20-th century when, to achieve compatibility with quantum mechanics and special relativity, the entire universe was reduced to potential forces and the analytic equations were "truncated" with the removal of the external terms.

In reality, Newton [3] *did not* propose his celebrated equations to be restricted to reversible forces derivable from a potential $F = -\partial V/\partial r$, but proposed them for the most general possible irreversible systems,

$$m_a \times \frac{\mathrm{d}v_{ka}}{\mathrm{d}t} = F_{ka}(t,r,v), \quad k = 1,2,3; \quad a = 1,2,\ldots,N, \tag{1.1}$$

where the conventional associative product of numbers, matrices, operators, etc., is denoted hereon with the symbol $\times$ so as to distinguish it from numerous other products needed later on.

Similarly, to be compatible with Newton's equations, Lagrange [4] and Hamilton [5] decomposed Newton's force into a potential and a nonpotential component, represented all potential forces with functions today known as the Lagrangian and the Hamiltonian, and represented nonpotential forces with external terms. Therefore the *true Lagrange and Hamilton equations* are given, respectively, by

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L(t,r,v)}{\partial v^k_a} - \frac{\partial L(t,r,v)}{\partial r^k_a} = F_{ak}(t,r,v), \tag{1.2a}$$

$$\frac{\mathrm{d}r^k_a}{\mathrm{d}t} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{\mathrm{d}p_{ak}}{\mathrm{d}t} = -\frac{\partial H(t,r,p)}{\partial r^k_a} + F_{ak}(t,r,p), \tag{1.2b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v^2_a - V(t,r,v), \quad H = \Sigma_a \frac{\mathbf{p}^2_a}{2 \times m_a} + V(t,r,p), \tag{1.2c}$$

$$V = U(t,r)_{ak} \times v^k_a + U_o(t,r), \quad F(t,r,v) = F(t,r,p/m). \tag{1.2d}$$

The analytic equations used throughout the 20-th century shall be referred to as the *truncated Lagrange and Hamilton equations* because of the removal of the external terms.

More recently, Santilli [6a] conducted comprehensive studies on the integrability conditions for the existence of a potential, or a Lagrangian, or a Hamiltonian, called *conditions of variational self-adjointness.* These study permit the rigorous decomposition of Newtonian forces into a component that is variationally self-adjoint (SA), thus being derivable from a potential, and a component that is not (NSA),

$$m_a \times \frac{\mathrm{d}v_{ka}}{\mathrm{d}t} = F^{\mathrm{SA}}_{ka}(t,r,v) + F^{\mathrm{NSA}}_{ka}(t,r,v). \tag{1.3}$$

Consequently, the true Lagrange and Hamilton equations can be more technically written as

$$\left[\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L(t,r,v)}{\partial v^k_a} - \frac{\partial L(t,r,v)}{\partial r^k_a}\right]^{\mathrm{SA}} = F^{\mathrm{NSA}}_{ak}(t,r,v), \tag{1.4a}$$

$$\left[\frac{\mathrm{d}r^k_a}{\mathrm{d}t} - \frac{\partial H(t,r,p)}{\partial p_{ak}}\right]^{\mathrm{SA}} = 0, \quad \left[\frac{\mathrm{d}p_{ak}}{\mathrm{d}t} + \frac{\partial H(t,r,p)}{\partial r^k_a}\right]^{\mathrm{SA}} = F^{\mathrm{NSA}}_{ak}(t,r,p). \tag{1.4b}$$

The *forgotten legacy of Newton, Lagrange and Hamilton is that irreversibility originates precisely in the nonpotential-NSA forces, that is, in the truncated external terms*, because all known potential-SA forces are reversible. The scientific imbalance of subsect. **1**˙1 is then due to the fact that no serious scientific study on irreversibility can be conducted with the truncated analytic equations and their operator counterpart, since these equations can only represent reversible systems.

Being born and educated in Italy, during his graduate studies the author had the opportunity of reading in the late 1960s the original works by Lagrange that were written mostly in Italian in Torino (where this paper has been written).

In this way, the author had the opportunity of verifying *Lagrange's analytic vision of representing irreversibility precisely via the external terms,* due to the impossibility of representing all possible physical events via the sole use of the Lagrangian, since the latter was conceived for the representation of reversible and potential events. As the reader can verify, Hamilton had, independently, the same vision.

Consequently, the truncation of the analytic equations caused the impossibility of a credible treatment of irreversibility at the purely classical level. The lack of a credible treatment of irreversibility then propagated at the subsequent quantum-mechanical and quantum field theoretical levels.

It then follows that *quantum mechanics cannot possibly be used for serious studies on irreversibility* because the discipline was constructed for the description of reversible quantized atomic orbits and not for irreversible systems.

While the validity of quantum mechanics for the arena of its original conception and construction is beyond scientific doubt, the assumption of quantum mechanics as the final operator theory for all conditions existing in the universe, such as orbits of particles in the core of a star, is outside the boundaries of serious science.

This establishes the need for the construction of a covering of quantum mechanics specifically conceived and constructed for quantitative treatments of irreversible systems.

1˙3. *Early representations of irreversible systems.* – As is well known, the brackets of the time evolution of the truncated analytic equations, the familiar *Poisson brackets,* characterize a *Lie algebra,* a feature that persists at the quantum level, thus establishing Lie algebras as the ultimate foundations of the physics of the 20-th century.

By contrast, the brackets of the time evolution of an observable $A(r, p)$ in phase space according to the analytic equations with external terms,

$$\frac{\mathrm{d}A}{\mathrm{d}t} = (A, H, F) = \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}} + \frac{\partial A}{\partial p_{ka}} \times F_{ka}, \tag{1.5}$$

cannot characterize any *algebra* as commonly understood in mathematics, because the brackets violate the right associative and scalar laws.

Therefore, the presence of external terms in the analytic equations causes not only the loss of *all* Lie algebras in the study of irreversibility, but actually the loss of all possible algebras.

To resolve this problem, the author initiated a long scientific journey following the reading of Lagrange's papers.

The original argument [7-9], still valid today, is to select analytic equations characterizing brackets in the time evolution that verify the following conditions:

1) Said brackets must verify the right and left associative and scalar laws to characterize an algebra.

2) Said brackets must not be invariant under time reversal as a necessary condition to represent irreversibility *ab initio*; and

3) the algebras characterized by said brackets must be a covering of Lie algebras as a necessary condition to characterize a covering of the truncated analytic equations, namely, as a condition for the selected representation of irreversibility to admit reversibility as a particular case.

Condition 1) requires that said brackets should be bilinear, *e.g.*, of the form (A, B) with basic algebraic axioms

$$(n \times A, B) = n \times (A, B), \quad (A, m \times B) = m \times (A, B); \quad n, m \in C, \tag{1.6a}$$

$$(A \times B, C) = A \times (B, C), \quad (A, B \times C) = (A, B) \times C. \tag{1.6b}$$

Condition 2) can be first realized by requiring that brackets (A, B) are not totally antisymmetric as the conventional Poisson brackets,

$$(A, B) \neq -(B, A), \tag{1.7}$$

because time reversal is realized via the use of Hermitean conjugation.

Condition 3) implies that brackets (A, B) characterize *Lie-admissible algebras* in the sense of Albert [10], with the operator form resulting to be also Jordan admissible according to the following

Definition 1.1: *A generally nonassociative algebra U with elements $a, b, c, \ldots$ and abstract product ab is said to be Lie-admissible when the attached algebra U^- characterized by the same vector space U equipped with the product $[a, b] = ab - ba$ verifies the Lie axioms*

$$[a, b] = -[b, a], \tag{1.8a}$$

$$[[a, b], c] + [[b, c], a] + [[c, b], a] = 0. \tag{1.8b}$$

Said generally nonassociative algebra U is said to be Jordan-admissible when the attached algebra U^+ characterized by the vector space U equipped with the product $\{a,b\} = ab+ba$ verifies the Jordan axioms

$$\{a,b\} = \{b,a\}, \tag{1.9a}$$
$$\{\{a,b\},a^2\} = \{a,\{b,a^2\}\}. \tag{1.9b}$$

In essence, the condition of Lie-admissibility requires that the brackets (A,B) contain a totally antisymmetric component with a residual totally symmetric part, thus admitting the decomposition

$$(A,B) = [A,B]^* + \{A,B\}^*, \tag{1.10}$$

where the * denotes expected generalizations of conventional brackets.

The antisymmetric brackets $[A,B]^*$ generally result to be Lie at both classical and operator levels. However, as illustrated below, for reasons yet unknown, the symmetric brackets $\{A,B\}^*$ always verify Jordan's first axiom (1.9a), but they generally violate the second axiom (1.9b) in their classical form and verify it only in their operator form.

After identifying the above lines, Santilli [9] proposed in 1967 the following *generalized analytic equations*:

$$\frac{\mathrm{d}r_a^k}{\mathrm{d}t} = \alpha \times \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{\mathrm{d}p_{ak}}{\mathrm{d}t} = -\beta \times \frac{\partial H(t,r,p)}{\partial r_a^k} \tag{1.11}$$

(where α and β are real non-null parameters), which equations are manifestly irreversible. The time evolution is then given by

$$\frac{\mathrm{d}A}{\mathrm{d}t} = (A,H) = = \alpha \times \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \beta \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}}, \tag{1.12}$$

whose brackets are manifestly Lie admissible because the attached antisymmetric brackets are proportional to the conventional Poisson brackets,

$$[A,B]^* = (A,B) - (B,A) = (\alpha+\beta) \times \left(\frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}} \right). \tag{1.13}$$

However, the attached symmetric brackets

$$[A,B]^* = (A,B) + (B,A) = (\alpha-\beta) \times \left(\frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} + \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}} \right) \tag{1.14}$$

do not characterize a Jordan algebra because they verify first axiom (1.9a) but violate the second axiom (1.9b) as the reader is encouraged to verify.

The above analytic equations characterize the time-rate of variation of the energy

$$\frac{\mathrm{d}H}{\mathrm{d}t} = (\alpha-\beta) \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}}. \tag{1.15}$$

Also in 1967, Santilli [7,8] proposed an operator counterpart of the preceding classical setting consisting in the first known *Lie-admissible* (p, q)*-parametric generalization of Heisenberg's equation* in the following infinitesimal and finite forms:

$$(1.16a) \qquad i \times \frac{\mathrm{d}A}{\mathrm{d}t} = (A, B) = p \times A \times H - q \times H \times A = \\ = m \times (A \times B - B \times A) + n \times (A \times B + B \times A),$$

$$(1.16b) \qquad A(t) = W(t) \times A(0) \times W^{\dagger}(t) = e^{i \times H \times q \times t} \times A(0) \times e^{-i \times t \times p \times H},$$

$$(1.16c) \qquad W \times W^{\dagger} \neq I,$$

where $p, q, p \pm q$ are non-null parameters, and

$$(1.17) \qquad m = p + q, \quad n = q - p\,.$$

Santilli's brackets (A, B), called a *parametric mutation* of Lie's brackets in the origional paper [7] for certain mathematical and physical reasons, are manifestly Lie admissible with attached antisymmetric part

$$(1.18) \qquad [A, B]^{*} = (A, B) - (B, A) = (p - q) \times [A, B].$$

The same brackets are also Jordan admissible as interested readers are encouraged to verify, thus illustrating the peculiar occurrence whereby symmetric brackets that are apparently similar are Jordan admissible only at the operator level.

Despite this limitation, the Jordan admissibility of the operator brackets establishes Jordan's dream on the physical applications of his algebras, although the latter occur for irreversible systems. As a matter of fact, the above Jordan admissibility establishes the impossibility for Jordan algebras to have applications at the purely quAntum level, due to its purely reversible, thus Lie character.

The time evolution of eqs. (1.16) is manifestly irreversible (for $p \neq q$) with nonconservation of the energy

$$(1.19) \qquad i \times \frac{\mathrm{d}H}{\mathrm{d}t} = (H, H) = (p - q) \times H \times H \neq 0.$$

Subsequent to papers [7-9], Santilli realized that the above formulations are not invariant under their own time evolution because eq. (1.16b) is manifestly *nonunitary.*

The application of nonunitary transforms to time evolution (1.16) then led to the proposal in memoir [11, 12] of 1978 of the following *Lie-admissible* (R, S)*-operator generalization of Heisenberg equations* in the following infinitesimal and finite forms:

$$(1.20a) \qquad i \times \frac{\mathrm{d}A}{\mathrm{d}t} = A < H - H > A = A \times R \times H - H \times S \times A = (A, H)^{*},$$

$$(1.20b) \qquad A(t) = W(t) \times A(0) \times W^{\dagger}(t) = (e^{i \times H \times S \times t}) \times A(0) \times (e^{-i \times t \times R \times H}), \\ W \times W^{\dagger} \neq I,$$

$$(1.20c) \qquad R = R(t, r, p, \psi, \partial\psi, ...) = S^{\dagger},$$

where R, S and $R \pm S$ are now nonsingular operators (or matrices), and eq. (1.20c) is a basic consistency condition explained later on.

Equations (1.20) are the *fundamental equations of hadronic mechanics* [12, 18]. Their basic brackets $(A, B)^*$, called an *operator mutation* of Lie's brackets, are manifestly Lie admissible and Jordan admissible with structure

$$(1.21a) \qquad (A,B)^* = A < B - B > A = A \times R \times B - B \times S \times A = \\ = (A \times T \times B - B \times T \times A) + (A \times Q \times B + B \times Q \times A),$$

$$(1.21b) \qquad T = R + S, \quad T = S - R.$$

The generalized classical equations proposed in refs. [11, 12] jointly with eqs. (1.20) are given in unified phase space notation by

$$(1.22a) \qquad \frac{\mathrm{d}b^\mu}{\mathrm{d}t} = S^{\mu\nu}(t,b) \times \frac{\partial H(b)}{\partial b^\nu},$$

$$(1.22b) \qquad b = (b^\mu) = (r^k, p_k), \quad \mu = 1, 2, ..., 6, \quad k = 1, 2, 3,$$

where the tensor $S^{\mu\nu}$ is Lie admissible (but not jointly Jordan admissible), namely, the attached antisymmetric tensor

$$(1.23) \qquad \Omega^{\mu\nu} = S^{\mu\nu} - S^{\nu\mu},$$

characterizes Birkhoff'a equations (see monograph [6b] for comprehensive studies), with solution

$$(1.24) \qquad S_{\mu\nu} = \frac{\partial R_\nu(t,b)}{\partial b^\mu}, \quad S^{\mu\nu} = [(S_{\alpha\beta})^{-1}]^{\mu\nu}.$$

The canonical particular case is recovered for the value $R = (R_\mu) = (p_k, 0)$ as the reader is encouraged to verify [6b, 11].

It is easy to see that the application of a nonunitary transform to the parametric equations (1.16) leads to the operator equations (1.20) and that the application of additional nonunitary transforms preserves their Lie-admissible and Jordan-admissible characters.

Consequently, fundamental equations (1.20) are "directly universal" in the sense of admitting as particular cases all possible brackets characterizing an algebra (universality) without the use of the transformation theory (direct universality).

1˙4. *Catastrophic inconsistencies of early irreversible formulations.* – Despite their direct universality, eqs. (1.20) *are not invariant under their own time evolution and, consequently, are afflicted by catastrophic inconsistencies* [29-37].

To clarify this crucial point, let us recall that the capability by quantum mechanics to predict the same numerical values under the same conditions at different times is given by the unitary character of Heisenberg's time evolution for Hermitean Hamiltonians,

$$(1.25a) \qquad A(t) = U(t) \times A(0) \times U^\dagger(t) = (e^{i \times H \times t}) \times A(0) \times (e^{-i \times t \times H}),$$

$$(1.25b) \qquad U \times U^\dagger = U^\dagger \times U = I, \quad H = H^\dagger,$$

that characterizes the following type of invariance of the Lie brackets:

$$(1.26a) \qquad U \times [A,B] \times U^\dagger = A' \times B' - B' \times A' = [A', B'],$$

$$(1.26b) \qquad A' = U \times A \times U^\dagger, \quad B' = U \times B \times U^\dagger,$$

namely, an invariance specifically referred to the preservation of the conventional associative product, $\times' \equiv \times$.

For the case of the broader Lie admissible equations (1.20) the situation is different because the Hamiltonian remains Hermitean, but the time evolution is nonunitary, in which case the application of the time evolution leads to the *new* brackets

$$
\begin{aligned}
(1.27a)\qquad & W \times (A,B)^{*} \times W^{\dagger} = A' <' B' - B' >' A' = \\
& = A' \times R' \times B' - B' \times S' \times S' \times A' = \\
& = A' \times (W^{\dagger -1} \times R \times W^{-1}) \times \\
& \times B' - B' \times (W^{\dagger -1} \times R \times W^{-1}) \times A' = (A', B')^{*\prime}, \\
(1.27b)\qquad & W \times W^{\dagger} \neq I, \quad H = H^{\dagger},
\end{aligned}
$$

namely, the transformed product remains Lie admissible due to its direct universality, but the product itself is altered, $R \to R' \neq R, S \to S' \neq S$.

As we shall see in sect. **4**, different values of the R and S operators characterize *different physical systems.* Consequently, the above change of values of the R and S operators caused by the time evolution of the theory cannot preserve numerical predictions in time.

It is instructive to verify that essentially the same occurrences hold for the classical Lie-admissible equations [11] due to the noncanonical character of the time evolution.

Without proof we, therefore, quote the following:

Theorem 1.2 [29]: *Whether Lie or Lie admissible, all classical theories possessing noncanonical time evolutions are afflicted by catastrophic mathematical and physical inconsistencies.*

Theorem 1.3 [29]: *All operator theories possessing a nonunitary time evolution formulated on conventional Hilbert spaces $\mathcal{H}$ over conventional fields of complex numbers C are afflicted by catastrophic mathematical and physical inconsistencies. In particular, said nonunitary theories*

1) do not possess invariant units of time, space, energy, etc., thus lacking physically meaningful application to measurements;

2) do not conserve Hermiticity in time, thus lacking physically meaningful observables;

3) do not possess unique and invariant numerical predictions;

4) generally violate probability and causality laws; and

5) violate the basic axioms of Galileo's and Einstein's relativities.

A comprehensive presentation of the above inconsistency theorems is available in Chapt. 1 of monograph [18c]. For the limited scope of this paper it is sufficient to indicate that the mathematical inconsistencies are rather serious. All mathematical formulations used in physics are based on fields that, in turn, are centrally dependent on the basic unit. However, noncanonical or nonunitary transformations do not preserve the basic unit by central assumption.

Hence, a given field at the initial time is no longer applicable at a subsequent time for all noncanonical or nonunitary theories, due to the lack of time invariance of the basic unit. In turn, the loss under the time evolution of the base field causes the catastrophic collapse of all mathematical formulations.

For example, the formulation of a noncanonical classical theory on the Euclidean space $\mathcal{E}$ over the field of real numbers R, or of a nonunitary operator theory on a Hilbert space $\mathcal{H}$ over the field of complex numbers C has no mathematical consistency, again, because of the lack of invariance of the basic unit of the fields (and, hence, of the field themselves) under the time evolution.

Conventional mathematics can at best be used for the treatment of noncanonical or nonunitary theories for the representation of systems at a fixed value of time without any possible dynamics. In this case no irreversibility can be consistently treated due to the need of a time evolution for its very manifestation.

The physical inconsistencies are equally catastrophic, as illustrated by occurrences 1) to 5) of Theorem 1.3 (see ref. [29] for details). It is sufficient here to recall that the basic unit of the three-dimensional Euclidean space $I = \mathrm{Diag}\,(1, 1, 1)$ represents in an abstract dimensionless form the assumed *measurement units, e.g.*, $I = \mathrm{Diag}\,(1\,\mathrm{cm}, 1\,\mathrm{cm}, 1\,\mathrm{cm})$. The loss of the measurement units under the time evolution then illustrates the catastrophic character of the inconsistencies due, *e.g.*, to the consequential lack of invariant numerical predictions.

Similarly, it is easy to prove that the condition of Hermiticity at the initial time,

$$(\langle\phi| \times H^\dagger) \times |\psi\rangle \equiv \langle\phi| \times (H \times |\psi\rangle), \quad H = H^\dagger, \tag{1.28}$$

is violated at subsequent times for theories with nonunitary time evolution when formulated on $\mathcal{H}$ over $\mathcal{C}$. This additional catastrophic inconsistency (known as *Lopez's lemma* [31, 32]), can be expressed by

$$\begin{aligned} (1.29a)\qquad & [\langle\psi| \times W^\dagger \times (W \times W^\dagger)^{-1} \times W \times H \times W^\dagger] \times W|\psi\rangle = \\ & = \langle\psi| \times W^\dagger \times [(W \times H \times W^\dagger) \times (W \times W^\dagger)^{-1} \times W \times |\psi\rangle] = \\ & = (\langle\hat{\psi} \times T \times H'^\dagger) \times |\hat{\psi}\rangle = \langle\hat{\psi}| \times (\hat{H} \times T \times |\hat{\psi}\rangle), \\ (1.29b)\qquad & |\hat{\psi}\rangle = W \times |\psi\rangle, \quad T = (W \times W^\dagger)^{-1} = T^\dagger, \\ (1.29c)\qquad & H'^\dagger = T^{-1} \times \hat{H} \times T \neq H. \end{aligned}$$

As a result, nonunitary theories treated with conventional mathematics do not admit physically meaningful observables.

Perhaps more insidious is the catastrophic inconsistency caused by the general violation of causality by theories with nonunitary time evolutions since the verification of causality laws by quantum mechanics is deeply linked to the unitarity of its time evolution, as well known.

By no means the above catastrophic inconsistencies solely apply to Santilli's early formulations of irreversibility, eqs. (1.20). In fact, due to the "direct universality" of theories (1.20), the same inconsistencies apply to a rather vast number of theories, such:

1) Dissipative nuclear theories [25] represented via an imaginary potential in non-Hermitean Hamiltonians,

$$H = H_0 = iV \neq H^\dagger\,, \tag{1.30}$$

lose all algebras in the brackets of their time evolution (requiring a bilinear product) in favor of the triple system,

$$i \times \;\mathrm{d}A/\mathrm{d}t = A \times H - H^\dagger \times A = [A, H, H^\dagger]. \tag{1.31}$$

This causes the loss of nuclear notions such as "protons and neutrons" as conventionally understood, *e.g.*, because the definition of their spin mandates the presence of a consistent algebra in the brackets of the time evolution.

2) Statistical theories with an external collision term C (see ref. [38] and literature quoted therein) and the equation of density

$$i \, \mathrm{d}\rho/\mathrm{d}t = \rho \odot H = [\rho, H] + C, \quad H = H^{\dagger} \tag{1.32}$$

violate the conditions for the product $\rho \odot H$ to characterize any algebra, as well as the existence of exponentiation to a finite transform, let alone violating the conditions of unitarity.

3) The so-called "q-deformations" of the Lie product introduced in 1989 (see, *e.g.*, [39-44] and very large literature quoted therein)

$$A \times B - q \times B \times A, \tag{1.33}$$

where q is a non-null scalar, are a trivial particular case of Santilli's (p, q)-deformations (1.16) introduced in 1967 [7] and, consequently, they are catastrophically inconsistent. As a matter of fact, when papers in q-deformations proliferated in the early 1990s, Santilli had abandoned the formulations he had initiated precisely in view of their catastrophic inconsistencies.

4) The so-called "k-deformations" [45-48] that are a relativistic version of the q-deformations, thus also being a particular case of Santilli's Lie-admissible structure.

5) The so-called "star deformations" [49] of the associative product

$$A \star B = A \times T \times B, \tag{1.34}$$

where T is fixed, and related generalized Lie product

$$A \star B - B \star A, \tag{1.35}$$

manifestly coincide with Santilli's Lie-isotopic algebras introduced in 1978 [11, 12], thus being manifestly nonunitary.

6) Deformed creation-annihilation operators theories [50, 51].

7) Nonunitary statistical theories [52].

8) Irreversible black-holes dynamics [53] with Santilli's Lie-admissible structure (1.21).

9) Noncanonical time theories [54-56].

10) Supersymmetric theories [57] with product

$$\begin{aligned} (A, B) &= [A, B] + \{A, B\} = \\ &= (A \times B - B \times A) + (A \times B + B \times A), \end{aligned} \tag{1.36}$$

are an evident particular case of Santilli's Lie-admissible product (1.21) with $T = W = I$.

11) String theories (see ref. [37] and literature quoted therein) generally have a noncanonical structure due to the inclusion of gravitation with additional catastrophic inconsistencies when including supersymmetries.

12) The so-called squeezed states theories [58, 59] due to their manifest nonunitary character.

13) All quantum groups (see, *e.g.*, refs. [60-62]) with a nonunitary structure.

14) Kac-Moody superalgebras [63] are also nonunitary and a particular case of Santilli's Lie-admissible algebra (1.10) with $T = I$ and Q a phase factor.

Numerous additional theories are also afflicted by the catastrophic inconsistencies of Theorem 1.3, such as quantum groups, quantum gravity, and other theories with nonunitary time evolution formulated on conventional Hilbert spaces over conventional fields the reader can easily identify in the literature.

All the above theories have a nonunitary structure formulated via conventional mathematics and, therefore, are afflicted by the catastrophic mathematical and physical inconsistencies of Theorem 1.3.

Additional generalized theories were attempted via *the relaxation of the linear character of quantum mechanics* [35]. These theories are essentially based on eigenvalue equations with the structure

$$H(t, r, p, |\psi\rangle) \times |\psi\rangle = E \times |\psi\rangle, \tag{1.37}$$

(*i.e.* H depends on the wave function).

Even though mathematically intriguing and possessing a seemingly unitary time evolution, these theories also possess rather serious physical drawbacks, such as: they violate the superposition principle necessary for composite systems like hadrons; they violate the fundamental Mackay imprimitivity theorem necessary for the applicability of Galileo's and Einstein's relativities and possess other drawbacks [18b] so serious to prevent consistent applications.

Yet another type of broader theory is *Weinberg's nonlinear theory* [64] with brackets of the type

$$\begin{aligned} &A \odot B - B \odot A = \\ &= \frac{\partial A}{\partial \psi} \times \frac{\partial B}{\partial \psi^\dagger} - \frac{\partial B}{\partial \psi} \times \frac{\partial A}{\partial \psi^\dagger}, \end{aligned} \tag{1.38}$$

where the product $A \odot B$ is *nonassociative.*

This theory violates Okubo's No-Quantization Theorem [30], prohibiting the use of nonassociative envelopes because of catastrophic physical consequences, such as the loss of equivalence between the Schrödinger and Heisenberg representations (the former remains associative, while the latter becomes nonassociative, thus resulting in inequivalence).

Weinberg's theory also suffers from the absence of any unit at all (that is, the absence of a quantity E such that $E \odot A = A \odot E = A$ for all possible A), with consequential inability to apply the theory to measurements, the loss of exponentiation to a finite transform (lack of Poincaré-Birkhoff-Witt theorem), and other inconsistencies studied in ref. [34].

These inconsistencies are not resolved by the adaptation of Weinberg's theory proposed by Jordan [65] as readers seriously interested in avoiding the publication of theories known to be inconsistent *ab initio* are encouraged to verify.

In conclusion, by the late 1970's Santilli had identified classical and operator generalized theories [11, 12] that were proved to be directly universal and include as trivial particular cases a plethora of simpler versions by various other authors.

However, all these theories subsequently resulted in being catastrophically inconsistent on mathematical and physical grounds because they are noninvariant under their time evolution when elaborated with conventional mathematics, thus mandating a reinspection of the foundations for the treatment of irreversibility.

1˙5. *Guide to the research.* – The first need for a serious study of irreversibility is the *identification of a new mathematics specifically constructed to recover the invariance of Hamiltonian theories at the classical and operator levels.*

As an example, classical Lie-admissible equations (1.15) and (1.22) are manifestly not derivable from a potential (because variationally non–self-adjoint [6a]), and, consequently, they do not admit a unique and ambiguous map into an operator form.

As a matter of fact, the lack of a universal representation of nonconservative forced via a variational principle is a main historical reason for the inability of the physics of the 20-th century to conduct quantitative studies on the origin of irreversibility.

As a result, a primary objective of the needed new mathematics is that of achieving a directly universal representation via a variational principle of all (sufficiently smooth) nonconservative and irreversible systems. The needed new mathematics will be presented in the next section. Our classical invariant treatment of irreversibility is presented in sect. **3** and its operator counterpart is presented in sect. **4**. Illustrative examples and the indication of intriguing open problems are presented in sect. **5**.

The second need for a serious study in the origin of irreversibility is the *inclusion of antimatter ab initio.*

Note that all the preceding theories were solely intended for the representation of matter and are inapplicable to antimatter for numerous reasons. For instance, classical equations (1.15) and (1.22) permit no differentiation at all between neutral matter and antimatter stars. Assuming that said equations admit a sort of operator map, the operator image of a classical representation of antimatter with the sole change of the sign of the charge (as solely done in the 20-th century) would be given by a "particle" with the wrong sign of the charge and definitely not by a charge conjugated "antiparticle".

In any case, one of the historical scientific imbalances of the 20-th century has been the treatment of matter at all possible levels, from Newtonian mechanics to quantum field theory, while antimatter was treated at the sole level of second quantization. In turn this imbalance has prohibited the initiation of the study whether a far away galaxy or quasar is made up of matter of antimatter (since such a study requires a consistent classical gravitational theory of antimatter with null total charge, thus without the use of the charge for the characterization of antimatter).

In view of these and other insufficiencies, the author had to conduct a separate laborious construction of a new mathematics specifically conceived for the treatment of antiparticle at the *classical* level in such a way to yield operator images equivalent to charge conjugated states.

This new mathematics is characterized by the application of the *isodual map* [22], here generically expressed by

$$(1.39)\quad Q(t,r,\psi,\partial\psi,...) \rightarrow Q^d(t^d,r^d,\psi^d,\partial^d\psi^d,...) = -Q^\dagger(-t^\dagger,-r^\dagger,-\psi^\dagger,-\partial^\dagger(-\psi^\dagger),...)$$

to the *totality* of the mathematical and physical formulations used for matter, resulting in this way in a new mathematics today known as *Santilli's isodual mathematics* [68-72], that includes the novel *isodual numbers, isodual fields, isodual spaces, isodual differential calculus, isodual functional analysis, isodual geometries, Lie-Santilli isodual theory, isodual symmetries, etc.*

The classical and operator isodual theory of antimatter cannot possibly be reviewed in this paper to avoid a prohibitive length. We must, therefore, refer the reader to the quoted literature.

The reader should be alerted that the restriction of the studies on the origin of irreversibility solely to matter, as done in the 20-th century, is insufficient and actually insidious because, as now known for grand unifications [22], certain basic insufficiencies emerge only when the study of antimatter is included *ab initio.*

The third need for a serious study of irreversibility is the *joint consideration of open, nonconservative and irreversible as well as closed, conservative and irreversible systems.*

As a matter of fact, some of the most relevant irreversible systems can be considered as isolated from the rest of the universe at the classical level (such as the study of the structure of Jupiter) as well as at the operator level (such as the study of the structure of a star), in which case these systems verify all ten conventional total conservation laws, while being intrinsically irreversible.

In view of such a need, *Santilli proposed his Lie-admissible formulations as a covering, not of conventional Lie formulation, but as a covering of the broader irreversible Lie-isotopic formulations* [11, 12]. The latter were originally based on the following operator equations:

$$(1.40a)\qquad i \times \frac{\mathrm{d}A}{\mathrm{d}t} = A \hat{\times} H - H \hat{\times} A = \\ = A \times T(t, r, \psi, \partial\psi, ...) \times H - H \times T(t, r, \psi, \partial\psi, \times A = [A, H]^*,$$

$$(1.40b)\qquad A(t) = W(t) \times A(0) \times W^\dagger(t) = (e^{i \times H \times T \times t}) \times A(0) \times (e^{-i \times t \times T \times H}),$$

$$(1.40c)\qquad W \times W^\dagger \neq I, \quad T = T^\dagger > 0, \quad H = H^\dagger,$$

with classical counterpart given by Birkhoff's equations [6b]

$$(1.41)\qquad \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\partial A}{\partial b^\mu} \times \Omega_{\mu\nu}(t, b) \times \frac{\partial H(b)}{\partial b^\nu} = [A, H]^*.$$

In both cases the brackets of the time evolution $[A, H]^*$ are totally antisymmetric, thus permitting the conservation law of the energy

$$(1.42)\qquad i \times \frac{\mathrm{d}H}{\mathrm{d}t} = [H, H]^* \equiv 0,$$

as well as all other conventional conservation laws [18], under a full representation of irreversibility given by

$$(1.43)\qquad T(t, r, \psi, \partial\psi, ...) \neq T(-t, r, \psi, \partial\psi, ...).$$

In particular, the generalized brackets $[A, B]^*$ verify the Lie axioms, although they have a nontrivial generalized structure (*e.g.*, due to the general noncommutativity of H and T) and they characterize a formulation, today known as *Lie-Santilli isotheory* [6b], [11,18,22-72] that includes *isoenveloping algebras, Lie-Santilli isoalgebras, Lie-Santilli isogroups, isorepresentation theory, isotransformation theory, etc.* [18, 66-72].

We cannot possibly review in this paper the latter formulations and are regrettably forced to refer the reader to the quoted literature. For the limited scope of this paper we merely indicate that closed irreversible conditions are generally obtained by restricting the Lie-admissible brackets to be antisymmetric, yet generalized. As an illustration, this condition is reached for the operator case by requiring that the generally different R and S operators coincide, are Hermitean and actually positive definite (for certain topological conditions required by the underlying isotopology), $R = S = T = T^\dagger > 0$. Irreversibility is then readily represented in an axiomatically consistent way via an explicit time dependence $T = T(t) \neq T(-t)$.

In summary, in this paper we shall consider: 1) Conventional, closed and reversible systems of particles represented with conventional symbols such as t, r, H, etc., conventional associative product $\times$ and conventional Lie theory. 2) Closed irreversible systems of particles represented with $\hat{}$ over conventional symbols, and the Lie-Santilli isotheory. 3) Open irreversible systems of particles moving forward in time represented with the upper index $^>$, and related Santilli's Lie-admissible theory. 4) Open irreversible systems of particles moving backward in time represented with the upper index $^<$, and related Santilli's Lie-admissible theory; and 5) the antiparticle counterpart of all preceding systems 1-4 represented with the superscript "d" denoting isoduality.

Needless to say, the polyhedric character of the problem as well as the complexity of each individual aspect, are such that, despite the decades of studies by the author, studies in the origin of irreversibility are at their infancy because so much remains to be done. It is hoped that this paper will stimulate young minds of any age to contribute to one of the ultimate frontiers of knowledge.

2. – Elements of Santilli genomathematics and its isodual

2˙1. *Genounits, genoproducts and their isoduals.* – In the author's view, there cannot be a truly new physical theory without a new mathematics, and there cannot be a truly new mathematics without new numbers. Therefore, the resolution of the problem of invariance for nonunitary theories required laborious efforts in the search of new numbers capable of representing irreversibility via their own axiomatic structure.

After a futile search in the mathematical literature and a number of failed attempts, Santilli proposed in refs. [11,12] of 1978 the construction of a *new mathematics* specifically conceived for the indicated task, whose number theory eventually reached mathematical maturity only in paper [13] of 1993, mathematical maturity for the new differential calculus in memoir [14] of 1996 and, finally, an invariant formulation of Lie-admissible formulations only in paper [15] of 1997.

The new Lie-admissible mathematics is today known as *Santilli genomathematics* [66-75], where the prefix "geno" suggested in the original proposal [11,12] is used in the Greek meaning of "inducting" new axioms (as compared to the prefix "iso" of the simpler realization denoting the preservation of the original axioms).

The basic idea is to lift the isounits of the Lie-isotopic theory [18] into a form that is still nowhere singular, but *non-Hermitean*, thus implying the existence of *two* different generalized units, today called *Santilli genounits* for the description of matter, that are generally written as [13]

$$\hat{I}^> = 1/\hat{T}^>, \quad {}^<\hat{I} = 1/{}^<\hat{T}, \tag{2.1a}$$

$$\hat{I}^> \neq {}^<\hat{I}, \quad \hat{I}^> = ({}^<\hat{I})^\dagger, \tag{2.1b}$$

with two additional *isodual genounits* for the description of antimatter [14]

$$(\hat{I}^{>})^d = -(\hat{I}^{>})^{\dagger} = -{}^{<}\hat{I} = -1/{}^{<}\hat{T}, \quad ({}^{<}\hat{I})^d = -\hat{I}^{>} = -1/\hat{T}^{>}. \tag{2.2}$$

Jointly, all conventional and/or isotopic products $A\hat{\times}B$ among generic quantities (numbers, vector fields, operators, etc.) are lifted in such a form to admit the genounits as the correct left and right units at all levels, *i.e.*

$$A > B = A \times \hat{T}^{>} \times B, \quad A > \hat{I}^{>} = \hat{I}^{>} > A = A, \tag{2.3a}$$

$$A < B = A \times^{<} \hat{T} \times B, \quad A <^{<} \hat{I} =^{<} \hat{I} < A = A, \tag{2.3b}$$

$$A >^d B = A \times \hat{T}^{>d} \times B, \quad A >^d \hat{I}^{>d} = \hat{I}^{>d} >^d A = A, \tag{2.3c}$$

$$A <^d B = A \times^{<} \hat{T}^d \times B, \quad A <^d {}^{<}\hat{I}^d = {}^{<}\hat{I}^d <^d A = A, \tag{2.3d}$$

for all elements A, B of the set considered.

As we shall see in sect. **3**, the above basic assumptions permit the representation of irreversibility with the most primitive possible quantities, the basic units and related products.

In particular, genounits permit an invariant representation of the external forces in Lagrange's and Hamilton's equations. As such, they are generally dependent on time, coordinates, momenta, wavefunctions and any other needed variable, *e.g.*, $\hat{I}^{>} = \hat{I}^{>}(t^{>}, r^{>}, p^{>}, \psi^{>}, \ldots)$.

The assumption of all *ordered product to the right* $>$ permits the representation of matter systems moving forward in time, while the assumption of all *ordered products to the left* $<$ can represent matter systems moving backward in time, with corresponding antimatter systems represented by the respective isodual ordered products $>^d = - >^{\dagger}$ and $<^d = - <>^{\dagger}$. Irreversibility is represented *ab initio* by the inequality $A > B \neq A < B$ for matter and $>^d \neq <^d$ for antimatter.

Note that the simpler isotopic cases are given by $\hat{I}^{>} =^{<} \hat{I} = \hat{I} = \hat{I}^{\dagger} > 0$ for matter and $\hat{I}^{>d} =^{<} \hat{I}^d = \hat{I}^d = \hat{I}^{d\dagger} < 0$ for antimatter.

In conclusion, the reader should be aware that genomathematics consists of *four* branches, the *forward and backward genomathematics for matter and their isoduals for antimatter*, each paid being interconnected by time reversal, the two pairs being interconnected by isodual map (1.39) that, as is well known [22], is equivalent to charge conjugation.

2·2. *Genonumbers, genofunctional analysis and their isoduals.* – Genomathematics began to reach maturity with the discovery made, apparently for the first time in paper [13] of 1993, that *the axioms of a field still hold under the ordering of all products to the right or, independently, to the left*.

This unexpected property permitted the formulation of *new numbers*, that can be best introduced as a generalization of the isonumbers [18], although they can also be independently presented as follows:

Definition 2.1 [13]: *Let* $F = F(a, +, \times)$ *be a field of characteristic zero as per Definitions 2.2.1 and 3.2.1. Santilli's forward genofields are rings* $\hat{F}^{>} = \hat{F}(\hat{a}^{>}, \hat{+}^{>}, \hat{\times}^{>})$ *with: elements*

$$\hat{a}^{>} = a \times \hat{I}^{>}, \tag{2.4}$$

where $a \in F$, $\hat{I}^> = 1/\hat{T}^>$ *is a non singular non-Hermitean quantity (number, matrix or operator) generally outside* F *and* $\times$ *is the ordinary product of* F*; the genosum* $\hat{+}^>$ *coincides with the ordinary sum* $+$,

$$\hat{a}^> \hat{+}^> \hat{b}^> \equiv \hat{a}^> + \hat{b}^>, \ \ \forall \hat{a}^>, \ \hat{b}^> \in \hat{F}^>, \tag{2.5}$$

consequently, the additive forward genounit $\hat{0}^> \in \hat{F}$ *coincides with the ordinary* $0 \in F$*; and the forward genoproduct* $>$ *is such that* $\hat{I}^>$ *is the right and left isounit of* $\hat{F}^>$,

$$\hat{I}^> \hat{\times} \hat{a}^> = \hat{a}^> > \hat{I}^> \equiv \hat{a}^>, \ \ \forall \hat{a}^> \in \hat{F}^>. \tag{2.6}$$

Santilli's forward genofields verify the following properties:

1) *For each element* $\hat{a}^> \in \hat{F}^>$ *there is an element* $\hat{a}^{>-\hat{1}^>}$, *called forward genoinverse, for which*

$$\hat{a}^> > \hat{a}^{>-\hat{I}^>} = \hat{I}^>, \ \ \forall \hat{a}^> \in \hat{F}^> . \tag{2.7}$$

2) *The genosum is commutative*

$$\hat{a}^> \hat{+}^> \hat{b}^> = \hat{b}^> \hat{+}^> \hat{a}^>, \tag{2.8}$$

and associative

$$(\hat{a}^> \hat{+}^> \hat{b}^>) +^> \hat{c}^> = \hat{a}^> \hat{+}^> (\hat{b}^> \hat{+}^> \hat{c}^>), \ \ \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F} . \tag{2.9}$$

3) *The forward genoproduct is associative*

$$\hat{a}^> > (\hat{b}^> > \hat{c}^>) = (\hat{a}^> > \hat{b}^>) > \hat{c}^>, \quad \forall \hat{a}^>, \hat{b}^>, \hat{c}^> \in \hat{F}^>; \tag{2.10}$$

but not necessarily commutative

$$\hat{a}^> > \hat{b}^> \neq \hat{b}^> > \hat{a}^> . \tag{2.11}$$

4) *The set* $\hat{F}^>$ *is closed under the genosum,*

$$\hat{a}^> \hat{+}^> \hat{b}^> = \hat{c}^> \in \hat{F}^>, \tag{2.12}$$

the forward genoproduct,

$$\hat{a}^> > \hat{b}^> = \hat{c}^> \in \hat{F}^>, \tag{2.13}$$

and right and left genodistributive compositions,

$$\hat{a}^> > (\hat{b}^> \hat{+}^> \hat{c}^>) = \hat{d}^> \in \hat{F}^>, \tag{2.14a}$$

$$(\hat{a}^> \hat{+}^> \hat{b}^>) > \hat{c}^> = \hat{d}^> \in \hat{F}^> \quad \forall \hat{a}^>, \hat{b}^>, \hat{c}^>, \hat{d}^> \in \hat{F}^> . \tag{2.14b}$$

5) *The set* $\hat{F}^>$ *verifies the right and left genodistributive law*

$$\hat{a}^> > (\hat{b}^> \hat{+}^> \hat{c}^>) = (\hat{a}^> \hat{+}^> \hat{b}^>) > \hat{c}^> = \hat{d}^>, \ \ \forall \hat{a}^>, \hat{b}^>, \hat{c}^>, \in \hat{F}^>. \tag{2.15}$$

In this way we have the forward genoreal numbers $\hat{R}^{>}$, the forward genocomplex numbers $\hat{C}^{>}$ and the forward genoquaternionic numbers $\hat{Q}C^{>}$ while the forward genooctonions $\hat{O}^{>}$ can indeed be formulated but they do not constitute genofields [14].

The backward genofields and the isodual forward and backward genofields are defined accordingly. Santilli's genofields are called of the first (second) kind when the genounit is (is not) an element of F.

The basic axiom-preserving character of genofields is illustrated by the following:

Lemma 2.1 [13]: *Genofields of first and second kind are fields (namely, they verify all axioms of a field).*

Note that the conventional product "2 multiplied by 3" is not necessarily equal to 6 because, for isodual numbers with unit -1 it is given by -6 [13].

The same product "2 multiplied by 3" is not necessarily equal to $+6$ or -6 because, for the case of isonumbers, it can also be equal to an arbitrary number, or a matrix or an integrodifferential operator depending on the assumed isounit [13].

In this section we point out that "2 multiplied by 3" can be ordered to the right or to the left, and the result is not only arbitrary, but yielding different numerical results for different orderings, $2 > 3 \neq 2 < 3$, all this by continuing to verify the axioms of a field per each order [13].

Once the forward and backward genofields have been identified, the various branches of genomathematics can be constructed via simple compatibility arguments.

For specific applications to irreversible processes there is first the need to construct the *genofunctional analysis,* studied in refs. [6,18] that we cannot review here for brevity. The reader is however warned that any elaboration of irreversible processes via Lie-admissible formulations based on conventional or isotopic functional analysis leads to catastrophic inconsistencies because it would be the same as elaborating quantum-mechanical calculations with genomathematics.

As an illustration, Theorems 1.2 and 1.3 of catastrophic inconsistencies are activated unless one uses the ordinary differential calculus is lifted, for ordinary motion in time of matter, into the following *forward genodifferentials and genoderivatives*:

$$\hat{\mathrm{d}}^{>} x = \hat{T}^{>}_{x} \times \mathrm{d}x, \quad \frac{\hat{\partial}^{>}}{\hat{\partial}^{>} x} = \hat{I}^{>}_{x} \times \frac{\partial}{\partial x}, \text{ etc.} \tag{2.16}$$

with corresponding backward and isodual expressions here ignored.

Similarly, all conventional functions and isofunctions, such as isosinus, isocosinus, isolog, etc., have to be lifted in the genoform

$$\hat{f}^{>}(x^{>}) = f(\hat{x}^{>}) \times \hat{I}^{>}, \tag{2.17}$$

where one should note the necessity of the multiplication by the genounit as a condition for the result to be in $\hat{R}^{>}$, $\hat{C}^{>}$, or $\hat{O}^{>}$.

2˙3. *Genogeometries and their isoduals*. – Particularly intriguing are the *genogeometries* [16] (see also monographs [18] for detailed treatments). They are best characterized by a simple genotopy of the isogeometries, although they can be independently defined.

As an illustration, the *Minkowski-Santilli forward genospace* $\hat{M}^{>}(\hat{x}^{>}, \hat{\eta}^{>}, \hat{R}^{>})$ over the genoreal $\hat{R}^{>}$ is characterized by the following space-time *genocoordinates, genometric* *and genoinvariant*:

$$(2.18a)\qquad \hat{x}^{>} = x\hat{I}^{>} = \{x^\mu\} \times \hat{I}^{>}, \quad \hat{\eta}^{>} = \hat{T}^{>} \times \eta, \quad \eta = \mathrm{Diag}\,(1,1,1,-1),$$

$$(2.18b)\qquad \hat{x}^{>2^{>}} = \hat{x}^{>\mu}\hat{\times}^{>}\hat{\eta}^{>}_{\mu\nu}\hat{\times}^{>}\hat{x}^{>\nu} = (x^\mu \times \hat{\eta}^{>}_{\mu\nu} \times x^\nu) \times \hat{I}^{>},$$

where the first expression of the genoinvariant is on genospaces while the second is its projection in our space-time.

Note that the Minkowski-Santilli genospace has, in general, an explicit dependence on space-time coordinates. Consequently, it is equipped with the entire formalism of the conventional Riemannian spaces covariant derivative, Christoffel's symbols, Bianchi identity, etc., only lifted from the isotopic into the genotopic form.

A most important feature is that *genospaces permit, apparently for the first time in scientific history, the representation of irreversibility directly via the basic genometric.* This is due to the fact that genometrics are nonsymmetric by conception, *e.g.*,

$$(2.19)\qquad \hat{\eta}^{>}_{\mu\nu} \quad\neq\quad \hat{\eta}^{>}_{\nu\mu}.$$

Consequently, *genotopies permit the lifting of conventional symmetric metrics into nonsymmetric forms,*

$$(2.20)\qquad \eta^{\mathrm{Minkow.}}_{\mathrm{Symm}} \quad\rightarrow\quad \hat{\eta}^{>\,\mathrm{Minkow.\text{-}Sant.}}_{\mathrm{NonSymm}}$$

Remarkably, *nonsymmetric metrics bare indeed permitted by the axioms of conventional spaces* as illustrated by the invariance

$$(2.21)\qquad (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \equiv [x^\mu \times (\hat{T}^{>} \times \eta_{\mu\nu}) \times x^\nu] \times T^{>-1} \equiv \equiv (x^\mu \times \hat{\eta}^{>}_{\mu\nu} \times x^\nu) \times \hat{I}^{>},$$

where $\hat{T}^{>}$ is assumed in this simple illustration to be a complex number.

Interested readers can then work out backward genogeometries and the isodual forward and backward genogeometries with their underlying genofunctional analysis.

This basic geometric feature was not discovered until recently because hidden where nobody looked for, in the basic unit. However, this basic geometric advance in the representation of irreversibility required the prior discovery of basically new numbers, Santilli's genonumbers with nonsymmetric unit and ordered multiplication [14].

2·4. *Santilli Lie-admissible theory and its isodual.* – Particularly important for irreversibility is the lifting of Lie's theory and Lie-Santilli's isotheories permitted by genomathematics, first identified by ref. [11] of 1978 (and then studied in various works, *e.g.*, [6, 18-22]) via the following genotopies:

1) The *forward and backward universal enveloping genoassociative algebra* $\hat{\xi}^{>}$, ${}^{<}\hat{\xi}$, with infinite-dimensional basis characterizing the *Poincaré-Birkhoff-Witt-Santilli genotheorem*

$$(2.22a)\qquad \hat{\xi}^{>} : \hat{I}^{>},\ \hat{X}_i,\ \hat{X}_i > \hat{X}_j,\ \hat{X}_i > \hat{X}_j > \hat{X}_k,\ \ldots,\ i \le j \le k,$$

$$(2.22b)\qquad {}^{<}\hat{\xi} : \hat{I},\ {}^{<}\hat{X}_i,\ \hat{X}_i < \hat{X}_j,\ \hat{X}_i < \hat{X}_j < \hat{X}_k,\ \ldots,\ i \le j \le k;$$

where the "hat" on the generators denotes their formulation on genospaces over genofields and their Hermiticity implies that $\hat{X}^> = ^< \hat{X} = \hat{X}$.

2) The *Lie-Santilli genoalgebras* characterized by the universal, jointly Lie- and Jordan-admissible brackets,

$$(2.23) \qquad ^<\hat{L}^> : (\hat{X}_i \hat{,} \hat{X}_j) = \hat{X}_i < \hat{X}_j - \hat{X}_j > \hat{X}_i = C_{ij}^k \times \hat{X}_k,$$

here formulated in an invariant form (see below).

3) The *Lie-Santilli genotransformation groups*

$$(2.24) \qquad ^<\hat{G}^> : \hat{A}(\hat{w}) = (\hat{e}^{\hat{i} \hat{\times} \hat{X} \hat{\times} \hat{w}})^> > \hat{A}(\hat{0}) <^< (\hat{e}^{-\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}}) = \\ = (e^{i \times \hat{X} \times \hat{T}^> \times w}) \times A(0) \times (e^{-i \times w \times ^<\hat{T} \times \hat{X}}),$$

where $\hat{w}^> \in \hat{R}^>$ are the *genoparameters;* the *genorepresentation theory*, etc.

2˙5. *Genosymmetries and nonconservation laws.* – The implications of the Santilli Lie-admissible theory are significant mathematically and physically. On mathematical grounds, the Lie-Santilli genoalgebras are "directly universal" and include as particular cases all known algebras, such as Lie, Jordan, flexible algebras, power associative algebras, quantum, algebras, supersymmetric algebras, Kac-Moody algebras, etc. (subsect. **1**˙5).

Moreover, when computed on the *genobimodule*

$$(2.25) \qquad ^<\hat{B}^> = ^< \hat{\xi} \times \hat{\xi}^>,$$

Lie-admissible algebras verify all Lie axioms, while deviations from Lie algebras emerge only in their *projection* on the conventional bimodule

$$(2.26) \qquad ^<B^> = ^< \xi \times \xi^>$$

of Lie's theory (see ref. [17] for the initiation of the genorepresentation theory of Lie-admissible algebras on bimodules).

This is due to the fact that the computation of the left action $A < B = A \times ^< \hat{T} \times B$ on $^<\hat{\xi}$ (that is, with respect to the genounit $^<\hat{I} = 1/^<\hat{T}$) yields the save value as the computation of the conventional product $A \times B$ on $^<\xi$ (that is, with respect to the trivial unit I), and the same occurs for the value of $A > B$ on $\hat{\xi}^>$.

The above occurrences explain the reason why the structure constant and the product in the r.h.s. of eq. (2.23) are those of a conventional Lie algebra.

In this way, thanks to genomathematics, *Lie algebras acquire a towering significance in view of the possibility of reducing all possible irreversible systems to primitive Lie axioms.*

The physical implications of the Lie-Santilli genotheory are equally far reaching. In fact, Noether's theorem on the reduction of reversible conservation laws to primitive Lie symmetries can be lifted to the *reduction, this time, of irreversible nonconservation laws to primitive Lie-Santilli genosymmetries.*

As a matter of fact, this reduction was the very first motivation for the construction of the genotheory in memoir [12] (see also monographs [6, 18-20]). The reader can then foresee similar liftings of all remaining physical aspects treated via Lie algebras.

The construction of the isodual Lie-Santilli genotheory is an instructive exercise for readers interested in learning the new methods.

3. – Lie-admissible classical mechanics for matter and its isodual for antimatter

3˙1. *Fundamental ordering assumption on irreversibility.* – Another reason for the inability during the 20-th century to conduct in-depth studies of irreversibility is the general belief that motion in time has only two directions, forward and backward (Eddington historical time arrows). In reality, motion in time admits *four* different forms, all essential for serious studies in irreversibility, given by: 1) *motion forward to future time* characterized by the forward genotime $\hat{t}^{>}$; 2) *motion backward to past time* characterized by the backward genotime $^{<}\hat{t}$; 3) *motion backward from future time* characterized by the isodual forward genotime $\hat{t}^{>d}$; and 4) *motion forward from past time* characterized by the isodual backward genotime $^{<}\hat{t}^{d}$.

It is at this point where the *necessity* of both time reversal and isoduality appears in its full light. In fact, time reversal is only applicable to matter and, being represented with Hermitean conjugation, permits the transition from motion forward to motion backward in time, $\hat{t}^{>} \rightarrow^{<} \hat{t} = (\hat{t}^{>})^{\dagger}$. If used alone, time reversal cannot identify all four directions of motions. The *only* additional conjugation known to this author that is applicable at all levels of study and is equivalent to charge conjugation, is isoduality [22].

The additional discovery of two complementary orderings of the product and related units, with corresponding isoduals versions, individually preserving the abstract axioms of a field has truly fundamental implications for irreversibility, since it permits the axiomatically consistent and invariant representation of irreversibility via the most ultimate and primitive axioms, those on the product and related unit. We, therefore, have the following:

Fundamental ordering assumption on irreversibility [15]: *Dynamical equations for motion forward in time of matter (antimatter) systems are characterized by genoproducts to the right and related genounits (their isoduals), while dynamical equations for the motion backward in time of matter (antimatter) are characterized by genoproducts to the left and related genounits (their isoduals) under the condition that said genoproducts and genounits are interconnected by time reversal expressible for generic quantities A, B with the relation*

$$(A > B)^{\dagger} = (A > \hat{T}^{>} \times B)^{\dagger} = B^{\dagger} \times (\hat{T}^{>})^{\dagger} \times A^{\dagger}, \tag{3.1}$$

namely,

$$\hat{T}^{>} = (^{<}\hat{T})^{\dagger} \tag{3.2}$$

thus recovering the fundamental complementary conditions (1.17) *or* (2.2)*.*

Unless otherwise specified, from now on physical and chemical expression for irreversible processes will have no meaning without the selection of one of the indicated two possible orderings.

3˙2. *Geno-Newtonian equations and their isoduals.* – Recall that, for the case of isotopies, the basic Newtonian systems are given by those admitting nonconservative internal forces restricted by certain constraints to verify total conservation laws called *closed non-Hamiltonian systems* [6b,18].

For the case of the genotopies under consideration here, the basic Newtonian systems are the conventional nonconservative systems without subsidiary constraints, known as *open non-Hamiltonian systems*, with generic expression (1.3), in which case irreversibility is entirely characterized by non–self-adjoint forces, since all conservative forces are reversible.

As is well known, the above equations are not derivable from any variational principle in the fixed frame of the observer [6], and this is the reason all conventional attempts for consistently quantizing nonconservative forces have failed for about one century. In turn, the lack of achievement of a consistent operator counterpart of nonconservative forces lead to the belief that they are illusory because they disappear at the particle level.

The studies presented in this paper have achieved the first and only physically consistent operator formulation of nonconservative forces known to the author. This goal was achieved by rewriting Newton's equations (1.3) into an identical form derivable from a variational principle. Still in turn, the latter objective was solely permitted by the novel genomathematics.

It is appropriate to recall that Newton was forced to discover new mathematics, the differential calculus, prior to being able to formulated his celebrated equations. Therefore, readers should not be surprised at the need for the new genodifferential calculus as a condition to represent all nonconservative Newton's systems from a variational principle.

Recall also from subsect. **3**˙1 that, contrary to popular beliefs, there exist *four* inequivalent directions of time. Consequently, time reversal alone cannot represent all these possible motions, and isoduality results to be the only known additional conjugation that, when combined with time reversal, can represent all possible time evolutions of both matter and antimatter.

The above setting implies the existence of four different new mechanics first formulated by Santilli in memoir [14] of 1996, and today known as *Newton-Santilli genomechanics,* namely

A) *Forward genomechanics* for the representation of forward motion of matter systems.

B) *Backward genomechanics* for the representation of the time reversal image of matter systems.

C) *Isodual backward genomechanics* for the representation of motion backward in time of antimatter systems, and

D) *isodual forward genomechanics* for the representation of time reversal antimatter systems.

These new mechanics are characterized by

1) Four different times, *forward and backward genotimes for matter systems and the backward and forward isodual genotimes for antimatter systems*

$$\hat{t}^{>} = t \times \hat{I}_t^{>}, \quad -\hat{t}^{>}, \quad \hat{t}^{>d}, \quad -\hat{t}^{>d}, \tag{3.3}$$

with (nowhere singular and non-Hermitean) *forward and backward time genounits and their isoduals* (Note that, to verify the condition of non-Hermiticity, the time genounits can be *complex valued.*),

$$\hat{I}_t^{>} = 1/\hat{T}_t^{>}, \quad -\hat{I}_t^{>}, \quad \hat{I}^{>d}_{\;t}, \quad -\hat{I}^{>d}_{\;t}. \tag{3.4}$$

2) The *forward and backward genocoordinates and their isoduals*

$$\hat{x}^{>} = x \times \hat{I}^{>}_{x}, \quad -\hat{x}^{>}, \quad \hat{x}^{>d}, \quad -\hat{x}^{>d}, \tag{3.5}$$

with (nowhere singular non-Hermitean) *coordinate genounit*

$$\hat{I}^{>}_{x} = 1/\hat{T}^{>}_{x}, \quad -\hat{I}^{>}_{x}, \quad \hat{I}^{>d}_{\ \ x}, \quad -\hat{I}^{>d}_{\ \ x}, \tag{3.6}$$

with *forward and backward coordinate genospace and their isoduals* $\hat{S}^{>}_{x}$, etc., and related *forward coordinate genofield and their isoduals* $\hat{R}^{>}_{x}$, etc.

3) The *forward and backward genospeeds and their isoduals*

$$\hat{v}^{>} = \hat{d}^{>}\hat{x}^{>}/\hat{d}^{>}\hat{t}^{>}, \quad -\hat{v}^{>}, \quad \hat{v}^{>d}, \quad -\hat{v}^{>d}, \tag{3.7}$$

with (nowhere singular and non-Hermitean) *speed genounit*

$$\hat{I}^{>}_{v} = 1/\hat{T}^{>}_{v}, \quad -\hat{I}^{>}_{v}, \quad \hat{I}^{>d}_{\ \ v}, \quad -\hat{I}^{>d}_{\ \ v}, \tag{3.8}$$

with related *forward speed backward genospaces and their isoduals* $\hat{S}^{>}_{v}$, etc., over *forward and backward speed genofields* $\hat{R}^{>}_{v}$, etc.

The above formalism then leads to the *forward genospace for matter systems*

$$\hat{S}^{>}_{\mathrm{tot}} = \hat{S}^{>}_{t} \times \hat{S}^{>}_{x} \times \hat{S}^{>}_{v}, \tag{3.9}$$

defined over the it forward genofield

$$\hat{R}^{>}_{tot} = \hat{R}^{>}_{t} \times \hat{R}^{>}_{x} \times \hat{R}^{>}_{v}, \tag{3.10}$$

with *total forward genounit*

$$\hat{I}^{>}_{tot} = \hat{I}^{>}_{t} \times \hat{I}^{>}_{x} \times \hat{I}^{>}_{v}, \tag{3.11}$$

and corresponding expressions for the remaining three spaces obtained via time reversal and isoduality.

The basic equations are given by

I) The *forward Newton-Santilli genoequations for matter systems* [14], formulated via the genodifferential calculus,

$$\hat{m}^{>}_{a} > \frac{\hat{d}^{>}\hat{v}^{>}_{ka}}{\hat{d}^{>}\hat{t}^{>}} = -\frac{\hat{\partial}^{>}\hat{V}^{>}}{\hat{\partial}^{>}\hat{x}^{>k}_{a}}. \tag{3.12}$$

II) The *backward genoequations for matter systems* that are characterized by time reversal of the preceding ones.

III) The *backward isodual genoequations for antimatter systems* that are characterized by the isodual map of the backward genoequations,

$$^{<}\hat{m}^{d}_{a} < \frac{^{<}\hat{d}^{d<}\hat{v}^{d}_{ka}}{^{<}\hat{d}^{d<}\hat{t}^{d}} = -\frac{^{<}\hat{\partial}^{d<}\hat{V}^{d}}{^{<}\hat{\partial}^{d<}\hat{x}^{dk}_{a}}; \tag{3.13}$$

IV) the *forward isodual genoequations for antimatter systems* characterized by time reversal of the preceding isodual equations.

Newton-Santilli genoequations (3.12) are "directly universal" for the representation of all possible (well-behaved) equations (1.3) in the frame of the observer because they admit a multiple infinity of solutions for any given non–self-adjoint force.

A simple representation occurs under the conditions assumed for simplicity,

$$N = \hat{I}_t^> = \hat{I}_v^> = 1, \tag{3.14}$$

for which eqs. (3.12) can be explicitly written as

$$\begin{aligned} \hat{m}^> > \frac{\hat{\mathrm{d}}^> \hat{v}^>}{\hat{\mathrm{d}}^> t} &= m \times \frac{\mathrm{d}\hat{v}^>}{\mathrm{d}t} = \\ = m \times \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}(x \times \hat{I}_x^>)}{\mathrm{d}t} &= m \times \frac{\mathrm{d}v}{\mathrm{d}t} \times \hat{I}_x^> + m \times x \times \frac{\mathrm{d}\hat{I}^>}{\mathrm{d}t} = \hat{I}_x^> \times \frac{\partial V}{\partial x}, \end{aligned} \tag{3.15}$$

from which we obtain the genorepresentation

$$F^{\mathrm{NSA}} = -m \times x \times \frac{1}{\hat{I}_x^>} \times \frac{\mathrm{d}\hat{I}_x^>}{\mathrm{d}t}, \tag{3.16}$$

that always admit solutions here left to the interested reader since in the next section we shall show a much simpler, universal, *algebraic* solution.

As one can see, in Newton's equations the nonpotential forces are part of the applied force, while in the Newton-Santilli genoequations nonpotential forces are represented by the genounits, or, equivalently, by the genodifferential calculus, in a way essentially similar to the case of isotopies.

The main difference between iso- and geno-equations is that isounits are Hermitean, thus implying the equivalence of forward and backward motions, while genounits are non-Hermitean, thus implying irreversibility.

Note also that the topology underlying Newton's equations is the conventional, Euclidean, local-differential topology which, as such, can only represent point particles.

By contrast, the topology underlying the Newton-Santilli genoequations is given by a genotopy of the isotopology [71], thus permitting for the representation of extended, nonspherical and deformable particles via forward genounits, *e.g.*, of the type

$$\hat{I}^> = \mathrm{Diag}\,(n_1^2, n_2^2, n_3^2, n_4^2) \times \Gamma^>(t, r, v, \ldots), \tag{3.17}$$

where $n_k^2,\ k = 1, 2, 3$ represents the semiaxes of an ellipsoid, n_4^2 represents the density of the medium in which motion occurs (with more general nondiagonal realizations here omitted for simplicity), and $\Gamma^>$ constitutes a nonsymmetric matrix representing non–self-adjoint forces, namely, the contact interactions among extended constituents occurring for the motion forward in time.

3·3. *Lie-admissible classical genomechanics and its isodual.* – In this subsection we show that, once rewritten in their identical genoform (3.12), Newton's equations for nonconservative systems are indeed derivable from a variational principle, with analytic equations possessing a Lie-admissible structure and Hamilton-Jacobi equations suitable for the first know consistent and unique operator map studied in the next section.

The most effective setting to introduce real-valued non-symmetric genounits is in the $6N$-dimensional *forward genospace (genocotangent bundle)* with local genocoordinates and their conjugates

$$\hat{a}^{>\mu} = a^{\rho} \times \hat{I}_{1\ \rho}^{>\mu}, \ \ (\hat{a}^{>\mu}) = \begin{pmatrix} \hat{x}_{\alpha}^{>k} \\ \hat{p}_{k\alpha}^{>} \end{pmatrix} \tag{3.18}$$

and

$$\hat{R}_{\mu}^{>} = R_{\rho} \times \hat{I}_{2\ \mu}^{>\rho}, \ \ (\hat{R}_{\mu}^{>}) = (\hat{p}_{k\alpha}, \hat{0}), \tag{3.19a}$$

$$\hat{I}_1^{>} = 1/\hat{T}_1^{>} = (\hat{I}_2^{>})^{\mathrm{T}} = (1/\hat{T}_2^{>})^{\mathrm{T}}, \tag{3.19b}$$

$$k = 1,2,3; \ \ \alpha = 1,2,\ldots,N; \ \ \ \mu, \ \ \rho = 1,2,\ldots 6N,$$

where the superscript "T" stands for transposed, and nowhere singular, real-valued and non-symmetric genometric and related invariant

$$\hat{\delta}^{>} = \hat{T}_{1\ 6N\times 6N}^{>} \delta_{6N\times 6N} \times \delta_{6N\times 6N}, \tag{3.20a}$$

$$\hat{a}^{>\mu} > \hat{R}_{\mu}^{>} = \hat{a}^{>\rho} \times \hat{T}_{1\ \rho}^{>\beta} \times \hat{R}_{\beta}^{>} = a^{\rho} \times \hat{I}_{2\ \rho}^{>\beta} \times R_{\beta}. \tag{3.20b}$$

In this case we have the following *genoaction principle* [14]:

$$\hat{\delta}^{>}\hat{\mathcal{A}}^{>} = \hat{\delta}^{>}\hat{\int}^{>} [\hat{R}_{\mu}^{>} >_a \hat{\mathrm{d}}^{>}\hat{a}^{>\mu} - \hat{H}^{>} >_t \hat{\mathrm{d}}^{>}\hat{t}^{>}] = \tag{3.21}$$

$$= \delta \int [R_{\mu} \times \hat{T}_{1\ \nu}^{>\mu}(t,x,p,\ldots) \times \mathrm{d}(a^{\beta} \times \hat{I}_{1\ \beta}^{>\nu}) - H \times \mathrm{d}t] = 0,$$

where the second expression is the projection on conventional spaces over conventional fields and we have assumed for simplicity that the time genounit is 1.

It is easy to prove that the above genoprinciple characterizes the following *forward Hamilton-Santilli genoequations* (originally proposed in ref. [11] of 1978 with conventional mathematics and in ref. [14] of 1996 with genomathematics (see also refs. [18-20]):

$$\hat{\omega}_{\mu\nu}^{>} > \frac{\hat{\mathrm{d}}^{>}\hat{a}^{\nu>}}{\hat{\mathrm{d}}^{>}\hat{t}^{>}} - \frac{\hat{\partial}^{>}\hat{H}^{>}(\hat{a}^{>})}{\hat{\partial}^{>}\hat{a}^{\mu>}} = \tag{3.22a}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} \mathrm{d}r/\mathrm{d}t \\ \mathrm{d}p/\mathrm{d}t \end{pmatrix} - \begin{pmatrix} 1 & K \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \partial H/\partial r \\ \partial H/\partial p \end{pmatrix} = 0,$$

$$\hat{\omega}^{>} = \Big(\frac{\hat{\partial}^{>} R_{\nu}^{>}}{\hat{\partial}^{>}\hat{a}^{\mu>}} - \frac{\hat{\partial}^{>}\hat{R}_{\mu}^{>}}{\hat{\partial}^{>}\hat{a}^{\nu>}} \Big) \times \hat{I}^{>} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \hat{I}^{>}, \tag{3.22b}$$

$$K = F^{\mathrm{NSA}}/(\partial H/\partial p), \tag{3.22c}$$

where one should note the "direct universality" of the simple algebraic solution (3.22c).

The time evolution of a quantity $\hat{A}^{>}(\hat{a}^{>})$ on the forward geno-phase-space can be written in terms of the following brackets:

$$\frac{\hat{\mathrm{d}}^{>}\hat{A}^{>}}{\hat{\mathrm{d}}^{>}t^{>}} = (\hat{A}^{>}, \hat{H}^{>}) = \frac{\hat{\partial}^{>}\hat{A}^{>}}{\hat{\partial}^{>}\hat{a}^{>\mu}} > \hat{\omega}^{\mu\nu>} > \frac{\hat{\partial}^{>}\hat{H}^{>}}{\hat{\partial}\hat{a}^{>\nu}} = \tag{3.23a}$$

$$= \frac{\partial \hat{A}^{>}}{\partial \hat{a}^{>\mu}} \times S^{\mu\nu} \times \frac{\partial \hat{H}^{>}}{\partial \hat{a}^{>\nu}} =$$

$$= \Big(\frac{\partial \hat{A}^{>}}{\partial \hat{r}^{>k}_{\alpha}} \times \frac{\partial \hat{H}^{>}}{\partial \hat{p}^{>}_{ka}} - \frac{\partial \hat{A}^{>}}{\partial \hat{p}^{>}_{ka}} \times \frac{\partial \hat{H}^{>}}{\partial \hat{r}^{>k}_{a}} \Big) + \frac{\partial \hat{A}^{>}}{\partial \hat{p}^{>}_{ka}} \times F^{\mathrm{NSA}}_{ka},$$

(3.23b) $$S^{>\mu\nu} = \omega^{\mu\rho} \times \hat{I}^{2}_{\rho}{}^{\mu}, \omega^{\mu\nu} = (||\omega_{\alpha\beta}||^{-1})^{\mu\nu},$$

where $\omega^{\mu\nu}$ is the conventional Lie tensor and, consequently, $S^{\mu\nu}$ is Lie-admissible in the sense of Albert [7].

As one can see, the important consequence of genomathematics and its genodifferential calculus is that of turning the triple system (A, H, F^{NSA}) of eqs. (1.5) in the bilinear form $(A\hat{,}B)$, thus characterizing a consistent algebra in the brackets of the time evolution.

This is the central purpose for which genomathematics was built (note that the multiplicative factors represented by K are fixed for each given system). The invariance of such a formulation will be proved shortly.

It is an instructive exercise for interested readers to prove that the brackets $(A\hat{,}B)$ are Lie admissible, although not Jordan admissible.

It is easy to verify that the above identical reformulation of Hamilton's historical time evolution correctly recovers the *time rate of variations of physical quantities* in general, and that of the energy in particular,

(3.24a) $$\frac{\mathrm{d}A^{>}}{\mathrm{d}t} = (A^{>}, H^{>}) = [\hat{A}^{>}, \hat{H}^{>}] + \frac{\partial \hat{A}^{>}}{\partial \hat{p}^{>}_{k\alpha}} \times F^{\mathrm{NSA}}_{k\alpha},$$

(3.24b) $$\frac{\mathrm{d}H}{\mathrm{d}t} = [\hat{H}^{>}, \hat{H}^{>}] + \frac{\partial \hat{H}^{>}}{\partial \hat{p}^{>}_{k\alpha}} \times F^{\mathrm{NSA}}_{ka} = v^{k}_{\alpha} \times F^{\mathrm{NSA}}_{ka}.$$

It is easy to show that genoaction principle (3.21) characterizes the following *Hamilton-Jacobi-Santilli genoequations* [14]:

(3.25a) $$\frac{\hat{\partial}^{>} \mathcal{A}^{>}}{\hat{\partial}^{>} \hat{t}^{>}} + \hat{H}^{>} = 0,$$

(3.25b) $$\Big(\frac{\hat{\partial}^{>} \mathcal{A}^{>}}{\hat{\partial}^{>} \hat{a}^{>\mu}} \Big) = \Big(\frac{\hat{\partial}^{>} \mathcal{A}^{>}}{\hat{\partial}^{>} x^{>k}_{a}}, \frac{\hat{\partial}^{>} \mathcal{A}^{>}}{\hat{\partial}^{>} p^{>}_{ka}} \Big) = (\hat{R}^{>}_{\mu}) = (\hat{p}^{>}_{ka}, \hat{0}),$$

which confirm the property (crucial for genoquantization as shown below) that the genoaction is indeed independent of the linear momentum.

Note the *direct universality* of the Lie-admissible equations for the representation of all infinitely possible Newton equations (1.3) (universality) directly in the fixed frame of the experimenter (direct universality).

Note also that, *at the abstract, realization-free level, Hamilton-Santilli genoequations coincide* with Hamilton's equations without external terms, yet represent those with external terms.

The latter are reformulated via genomathematics as the only known way to achieve invariance and derivability from a variational principle while admitting a consistent algebra in the brackets of the time evolution [38].

Therefore, Hamilton-Santilli genoequations (3.66) are indeed irreversible for all possible reversible Hamiltonians, as desired. The origin of irreversibility rests in the contact nonpotential forces F^{NSA} according to Lagrange's and Hamilton's teaching that is merely reformulated in an invariant way.

The above Lie-admissible mechanics requires, for completeness, *three* additional formulations, the *backward genomechanics* for the description of *matter moving backward in time,* and the isoduals of both the forward and backward mechanics for the description of *antimatter.*

The construction of these additional mechanics is left to the interested reader for brevity.

4. – Lie-admissible operator mechanics for matter and its isodual for antimatter

4˙1. *Basic dynamical equations.* – A simple genotopy of the naive or symplectic quantization applied to eqs. (3.24) yields the *Lie-admissible branch of hadronic mechanics* [18] comprising four different formulations, the *forward and backward genomechanics for matter and their isoduals for antimatter.* The forward genomechanics for matter is characterized by the following main topics:

1) The nowhere singular (thus everywhere invertible) non-Hermitean *forward genounit* for the representation of all effects causing irreversibility, such as contact nonpotential interactions among extended particles, etc. (see the subsequent sections for various realizations)

$$\hat{I}^{>} = 1/\hat{T}^{>} \neq (\hat{I}^{>})^{\dagger}, \tag{4.1}$$

with corresponding ordered product and genoreal $\hat{R}^{>}$ and genocomplex $\hat{C}^{>}$ genofields.

2) The *forward genotopic Hilbert space* $\hat{\mathcal{H}}^{>}$ with *forward genostates* $|\hat{\psi}^{>}>$ and *forward genoinner product*

$$<^{<}\hat{\psi}| > |\hat{\psi}^{>}> \times\hat{I}^{>} = <^{<}\hat{\psi}| \times \hat{T}^{>} \times |\hat{\psi}^{>}> \times\hat{I}^{>} \in \hat{C}^{>}, \tag{4.2}$$

and fundamental property

$$\hat{I}^{>} > |\hat{\psi}^{>}> = |\hat{\psi}^{>}>, \tag{4.3}$$

holding under the condition that $\hat{I}^{>}$ is indeed the correct unit for motion forward in time, and *forward genounitary transforms*

$$\hat{U}^{>} > (^{<}\hat{U})^{\dagger>} = (^{<}\hat{U})^{\dagger>} > \hat{U}^{>} = \hat{I}^{>}\,. \tag{4.4}$$

3) The fundamental Lie-admissible equations, first proposed in ref. [12] of 1974 (p. 783, eqs. (4.18.16)) as the foundations of hadronic mechanics, formulated on conventional spaces over conventional fields, and first formulated in refs. [14, 18] of 1996 on genospaces and genodifferential calculus on genofields, today's known as *Heisenberg-Santilli genoequations*, that can be written in the finite form

$$\begin{aligned}\hat{A}(\hat{t}) &= \hat{U}^{>} > \hat{A}(0) <^{<} \hat{U} = (\hat{e}_{>}^{\hat{i}\hat{\times}\hat{H}\hat{\times}\hat{t}}) > \hat{A}(\hat{0}) < ({}_{<}\hat{e}^{-\hat{i}\hat{\times}\hat{t}\hat{\times}\hat{H}}) = \\ &= (e^{i\times\hat{H}\times\hat{T}^{>}\times t}) \times A(0) \times (e^{-i\times t\times^{<}\hat{T}\times\hat{H}}),\end{aligned} \tag{4.5}$$

with the corresponding infinitesimal version

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A}\hat{;}\hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} = \tag{4.6}$$
$$= \hat{A} \times^{<} \hat{T}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \dots.) \times \hat{H} - \hat{H} \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \dots) \times \hat{A},$$

where there is no time arrow, since Heisenberg's equations are computed at a fixed time.

4) The equivalent *Schrödinger-Santilli genoequations*, first suggested in the original proposal [12] to build hadronic mechanics (see also refs. [17, 23, 24]), formulated via conventional mathematics and in refs. [14,18] via genomathematics, that can be written as

$$\hat{i}^{>} > \frac{\hat{\partial}^{>}}{\hat{\partial}^{>}\hat{t}^{>}}|\hat{\psi}^{>} >= \hat{H}^{>} > |\hat{\psi}^{>} >= \tag{4.7}$$
$$= \hat{H}(\hat{r}, \hat{v}) \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \hat{\partial}\hat{\psi} \dots) \times |\hat{\psi}^{>} >= E^{>} > |\psi^{>} >,$$

where the time orderings in the second term are ignored for simplicity of notation.

5) The *forward genomomentum* that escaped identification for two decades and was finally identified thanks to the genodifferential calculus in ref. [14] of 1996

$$\hat{p}^{>}_{k} > |\hat{\psi}^{>} >= -\hat{i}^{>} > \hat{\partial}^{>}_{k}|\hat{\psi}^{>} >= -i \times \hat{I}^{>\,i}_{k} \times \partial_i|\hat{\psi}^{>} > . \tag{4.8}$$

6) The *fundamental genocommutation rules* also first identified in ref. [14],

$$(\hat{r}^{i}\hat{;}\hat{p}_j) = i \times \delta^i_j \times \hat{I}^{>}, \ \ (\hat{r}^{i}\hat{;}\hat{r}^{j}) = (\hat{p}_i\hat{;}\hat{p}_j) = 0. \tag{4.9}$$

7) The *genoexpectation values* of an observable for the forward motion $\hat{A}^{>}$ [14, 19]

$$\frac{<^{<} \hat{\psi}| > \hat{A}^{>} > |\hat{\psi}^{>} >}{<^{<} \hat{\psi}| > |\hat{\psi}^{>} >} \times \hat{I}^{>} \in \hat{C}^{>}, \tag{4.10}$$

under which the genoexpectation values of the genounit recovers the conventional Planck's unit as in the isotopic case,

$$\frac{< \hat{\psi}| > \hat{I}^{>} > |\hat{\psi} >}{< \hat{\psi}| > |\hat{\psi} >} = I. \tag{4.11}$$

The following comments are now in order. Note first in the genoaction principle the crucial independence of isoaction $\hat{\mathcal{A}}^{>}$ in form the linear momentum, as expressed by the Hamilton-Jacobi-Santilli genoequations (3.25). Such independence assures that genoquantization yields a genowavefunction solely dependent on time and coordinates, $\hat{\psi}^{>} = \hat{\psi}^{>}(t, r)$.

Other geno-Hamiltonian mechanics studied previously [7] do not verify such a condition, thus implying genowavefunctions with an explicit dependence also on linear momenta, $\hat{\psi}^{>} = \hat{\psi}^{>}(t, r, p)$ that violate the abstract identity of quantum and hadronic mechanics whose treatment in any case is beyond our operator knowledge at this writing.

Note that *forward geno-Hermiticity coincides with conventional Hermiticity.* As a result, *all quantities that are observables for quantum mechanics remain observables for the above genomechanics.*

However, unlike quantum mechanics, physical quantities are generally *nonconserved,* as it must be the case for the energy,

$$\hat{i}^{>} > \frac{\hat{\mathrm{d}}^{>} \hat{H}^{>}}{\hat{\mathrm{d}}^{>} \hat{t}^{>}} = \hat{H} \times ({}^{<}\hat{T} - \hat{T}^{>}) \times \hat{H} \neq 0. \tag{4.12}$$

Therefore, *the genotopic branch of hadronic mechanics is the only known operator formulation permitting nonconserved quantities to be Hermitean as a necessary condition to be observability.*

Other formulation attempt to represent nonconservation, *e.g.*, by adding an "imaginary potential" to the Hamiltonian, as it is often done in nuclear physics [25]. In this case the Hamiltonian is non-Hermitean and, consequently, the nonconservation of the energy cannot be an observable.

Besides, said "nonconservative models" with non-Hermitean Hamiltonians are nonunitary and are formulated on conventional spaces over conventional fields, thus suffering all the catastrophic inconsistencies of Theorem 1.3.

We should stress the representation of irreversibility and nonconservation beginning with the most primitive quantity, the unit and related product. *Closed irreversible systems* are characterized by the Lie-isotopic subcase in which

$$\hat{i} \hat{\times} \frac{\hat{\mathrm{d}} \hat{A}}{\hat{\mathrm{d}} \hat{t}} = [\hat{A} \hat{,} \hat{H}] = \hat{A} \times \hat{T}(t, \dots) \times \hat{H} - \hat{H} \times \hat{T}(t, \dots) \times \hat{A}, \tag{4.13a}$$

$${}^{<}\hat{T}(t, \dots) = \hat{T}^{>}(t, \dots) = \hat{T}(t, \dots) = \hat{T}^{\dagger}(t, \dots) \neq \hat{T}(-t, \dots), \tag{4.13b}$$

for which the Hamiltonian is manifestly conserved. Nevertheless the system is manifestly irreversible. Note also the first and only known observability of the Hamiltonian (due to its iso-Hermiticity) under irreversibility.

As one can see, brackets (A, B) of eqs. (4.6) are jointly Lie and Jordan admissible.

Note also that finite genotransforms (4.5) verify the condition of genohermiticity, eq. (4.4).

We should finally mention that, as was the case for isotheories, *genotheories are also admitted by the abstract axioms of quantum mechanics, thus providing a broader realization.* This can be seen, *e.g.*, from the invariance under a complex number C

$$< \psi | x | \psi > \times I = < \psi | x C^{-1} \times | \psi > \times (C \times I) = < \psi | > | \psi > \times I^{>}. \tag{4.14}$$

Consequently, *genomechanics provide another explicit and concrete realization of "hidden variables"* [26]*, thus constituting another "completion" of quantum mechanics in the E-P-R sense* [27]. For the studies of these aspects we refer the interested reader to ref. [28].

The above formulation must be completed with three additional Lie-admissible formulations, the backward formulation for matter under time reversal and the two additional isodual formulations for antimatter. Their study is left to the interested reader for brevity.

4·2. *Simple construction of Lie-admissible theories*. – As was the case for the isotopies, a simple method has been identified in ref. [44] for the construction of Lie-admissible (geno-) theories from any given conventional, classical or quantum formulation. It consists in *identifying the genounits as the product of two different nonunitary transforms,*

$$\hat{I}^{>} = ({}^{<}\hat{I})^{\dagger} = U \times W^{\dagger}, \quad {}^{<}\hat{I} = W \times U^{\dagger}, \tag{4.15a}$$

$$U \times U^{\dagger} \neq 1, \quad W \times W^{\dagger} \neq 1, \quad U \times W^{\dagger} = \hat{I}^{>}, \tag{4.15b}$$

and subjecting the totality of quantities and their operations of conventional models to said dual transforms,

$$I \rightarrow \hat{I}^{>} = U \times I \times W^{\dagger}, \quad I \rightarrow^{<} \hat{I} = W \times I \times U^{\dagger}, \tag{4.16a}$$

$$a \rightarrow \hat{a}^{>} = U \times a \times W^{\dagger} = a \times \hat{I}^{>}, \tag{4.16b}$$

$$a \rightarrow^{<} \hat{a} = W \times a \times U^{\dagger} =^{<} \hat{I} \times a, \tag{4.16c}$$

$$\begin{aligned} a \times b \rightarrow \hat{a}^{>} > \hat{b}^{>} = U \times (a \times b) \times W^{>} = \\ = (U \times a \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times b \times W^{\dagger}), \end{aligned} \tag{4.16d}$$

$$\partial/\partial x \rightarrow \hat{\partial}^{>}/\hat{\partial}^{>}\hat{x}^{>} = U \times (\partial/\partial x) \times W^{\dagger} = \hat{I}^{>} \times (\partial/\partial x), \tag{4.16e}$$

$$< \psi| \times |\psi > \rightarrow < ^{<} \psi| > |\psi^{>} > = U \times (< \psi| \times |\psi >) \times W^{\dagger}, \tag{4.16f}$$

$$\begin{aligned} H \times |\psi > \rightarrow \hat{H}^{>} > |\psi^{>} > = \\ = (U \times H \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times \psi > W^{\dagger}), \text{ etc.} \end{aligned} \tag{4.16g}$$

As a result, any given conventional, classical or quantum model can be easily lifted into the genotopic form.

Note that the above construction implies that *all conventional physical quantities acquire a well-defined direction of time.* For instance, the correct genotopic formulation of energy, linear momentum, etc., is given by

$$\hat{H}^{>} = U \times H \times W^{\dagger}, \;\; \hat{p}^{>} = U \times p \times W^{>}, \text{ etc.} \tag{4.17}$$

In fact, under irreversibility, the value of a nonconserved energy at a given time t for motion forward in time is generally different than the corresponding value of the energy for $-t$ for motion backward in past times.

This explains the reason for having represented in this section energy, momentum and other quantities with their arrow of time $>$. Such an arrow can indeed be omitted for notational simplicity, but only after the understanding of its existence.

Note finally that a conventional, one-dimensional, unitary Lie transformation group with Hermitean generator X and parameter w can be transformed into a covering Lie-admissible group via the following nonunitary transform:

$$Q(w) \times Q^{\dagger}(w) = Q^{\dagger}(w) \times Q(w) = I, \; w \in R, \tag{4.18a}$$

$$U \times U^{\dagger} \neq I, \quad W \times W^{\dagger} \neq 1, \tag{4.18b}$$

$$\begin{aligned} A(w) = Q(w) \times A(0) \times Q^{\dagger}(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X} \rightarrow \\ \rightarrow U \times (e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}) \times U^{\dagger} = \\ \equiv [U \times (e^{X \times w \times i}) \times W^{\dagger} \times (U \times W^{\dagger})^{-1} \times A \times A(0) \times \\ \times U^{\dagger} \times (W \times U^{\dagger})^{-1} \times [W \times (e^{-i \times w \times X}) \times U^{\dagger}] = \end{aligned} \tag{4.18c}$$

$$= (e^{i\times X\times X})^{>} > A(0) <^{<} (e^{-1\times w\times X}) = \hat{U}^{>} > A(0) <^{<} \hat{U},$$

which confirm the property of subsect. **4**˙2, namely, that under the necessary mathematics *the Lie-admissible theory is indeed admitted by the abstract Lie axioms, and it is a realization of the latter broader than the isotopic form.*

4˙3. *Invariance of Lie-admissible theories.* – Recall that a fundamental axiomatic feature of quantum mechanics is the invariance under time evolution of all numerical predictions and physical laws, which invariance is due to the *unitary structure* of the theory.

However, quantum mechanics is reversible and can only represent in a scientific way beyond academic beliefs reversible systems verifying total conservation laws due to the antisymmetric character of the brackets of the time evolution.

As indicated earlier, the representation of irreversibility and nonconservation requires theories with a *nonunitary structure*. However, the latter are afflicted by the catastrophic inconsistencies of Theorem 1.3.

The only resolution of such a basic impasse known to the author has been the achievement of invariance under nonunitarity and irreversibility via the use of genomathematics, provided that such genomathematics is applied to the *totality* of the formalism to avoid evident inconsistencies caused by mixing different mathematics for the selected physical problem.

Let us note that, due to decades of protracted use, it is easy to predict that physicists and mathematicians may be tempted to treat the Lie-admissible branch of hadronic mechanics with conventional mathematics, whether in part or in full. Such a posture would be equivalent, for instance, to the elaboration of the spectral emission of the hydrogen atom with the genodifferential calculus, resulting in an evident nonscientific setting.

Such an invariance was first achieved by Santilli in ref. [15] of 1997 and can be illustrated by reformulating any given nonunitary transform in the *genounitary form*

$$U = \hat{U} \times \hat{T}^{>1/2}, W = \hat{W} \times \hat{T}^{>1/2}, \tag{4.19a}$$

$$U \times W^{\dagger} = \hat{U} > \hat{W}^{\dagger} = \hat{W}^{\dagger} > \hat{U} = \hat{I}^{>} = 1/\hat{T}^{>}, \tag{4.19b}$$

and then showing that genounits, genoproducts, genoexponentiation, etc., are indeed invariant under the above genounitary transform in exactly the same way as conventional units, products, exponentiations, etc. are invariant under unitary transforms,

$$\hat{I}^{>} \rightarrow \hat{I}^{>'} = \hat{U} > \hat{I}^{>} > \hat{W}^{\dagger} = \hat{I}^{>}, \tag{4.20a}$$

$$\begin{aligned} &\hat{A} > \hat{B} \rightarrow \hat{U} > (A > B) > \hat{W}^{\dagger} = \\ &= (\hat{U} \times \hat{T}^{>} \times A \times T^{>} \times \hat{W}^{\dagger}) \times (\hat{T}^{>} \times W^{\dagger})^{-1} \times \hat{T}^{>} \times \\ &\times (\hat{U} \times \hat{T}^{>})^{-1} \times (\hat{U} \times T^{>} \times \hat{A} \times T^{>} \times \hat{W}^{>}) = \\ &= \hat{A}' \times (\hat{U} \times \hat{W}^{\dagger})^{-1} \times \hat{B} = \hat{A}' \times \hat{T}^{>} \times B' = \hat{A}' > \hat{B}', \text{ etc.} \end{aligned} \tag{4.20b}$$

from which all remaining invariances follow, thus resolving the catastrophic inconsistencies of Theorem 1.3.

Note the *numerical invariances of the genounit* $\hat{I}^{>} \rightarrow \hat{I}^{>'} \equiv \hat{I}^{>}$, *of the genotopic element* $\hat{T}^{>} \rightarrow \hat{T}^{>'} \equiv \hat{T}^{>}$, *and of the genoproduct* $> \rightarrow >' \equiv >$ that are necessary to have invariant numerical predictions.

5. – Applications

5˙1. *Lie-admissible treatment of a particle with linear dissipative force*. – In this section we present a variety of classical and operator representations of nonconservative systems by omitting hereon for simplicity of notations all "hats" on quantities (denoting isotopies not considered in this section), omitting the symbol $\times$ to denote the conventional (associative) multiplication, but preserving the forward (backward) symbols $>$ ($<$) denoting forward (backward) motion in time for quantities and products.

Let us begin with a classical and operator representation of the simplest possible dissipative system, a massive particle moving within a physical medium, and being subjected to to a linear, velocity-dependent resistive force

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = F^{\mathrm{NSA}} = -kv, \tag{5.1}$$

for which we have the familiar *variation (dissipation) of the energy*

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv^2\right) = -kv^2. \tag{5.2}$$

Progressively more complex examples will be considered below.

The representations of system (5.1) via the *Newton-Santilli genoequations* (3.12) is given by

$$m^> > \frac{\mathrm{d}^> v^>}{\mathrm{d}^> t^>} = 0. \tag{5.3}$$

As indicated in sect. **3**, the representation requires the selection of *three* generally different genounits, $I_t^>, I_r^>, I_v^>$. Due to the simplicity of the case and the velocity dependence of the applied force, the simplest possible solution is given by

$$I_t^> = I_r^> = 1, \quad I_v^>(t) = e^{\frac{k\times t}{m}} = 1/T_v^>(t) > 0, \tag{5.4a}$$

$$m^> > \frac{d^> v^>}{d^> t^>} = m\frac{\mathrm{d}(vI_v^>)}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}t}I^> + kv\frac{\mathrm{d}I_v^>}{\mathrm{d}t} = 0\,. \tag{5.4b}$$

The representation with *Hamilton-Santilli genoequations* (3.22) is also straightforward and can be written in disjoint $r^>$ and $p^>$ notations

$$H^> = \frac{{p^>}^{2>}}{2^> > m^>} = \frac{p^2}{2m}I_p^>, \tag{5.5a}$$

$$v^> = \frac{\partial^> H^>}{\partial^> p^>} = \frac{p^>}{m}, \quad \frac{d^> p^>}{d^> t^>} = -\frac{\partial^> H^>}{\partial^> r^>} = 0. \tag{5.5b}$$

The last equation then reproduces equation of motion (5.1) identically under assumptions (5.4a).

The above case is instructive because the representation is achieved via the genoderivatives (subsect. **2**˙2). However, the representation exhibits no algebra in the time evolution. Therefore, we seek an alternative representation in which the dissipation is characterized by the Lie-admissible algebra, rather by the differential calculus.

This alternative representation is provided by the Hamilton-Santilli genoequations (3.22) in the unified notation $a^{>} = (r^{>k}, p_k^{>})$ that become for the case at hand

$$\text{(5.6)}\quad \frac{\mathrm{d}a^{>\mu}}{\mathrm{d}t} = \begin{pmatrix} \mathrm{d}r^{>}/\mathrm{d}t \\ \mathrm{d}p^{>}/\mathrm{d}t \end{pmatrix} = S^{>\mu\nu}\frac{\partial^{>}H^{>}}{\partial^{>}a^{>\nu}} = \begin{pmatrix} 0 & -1 \\ 1 & \frac{(-kv}{(\partial H/\partial p))} \end{pmatrix} \begin{pmatrix} \partial^{>}H^{>}/\partial^{>}r^{>} \\ \partial^{>}H^{>}/\partial^{>}p^{>} \end{pmatrix},$$

under which we have the genoequations

$$\text{(5.7)}\quad \frac{\mathrm{d}r^{>}}{\mathrm{d}t} = \frac{\partial^{>}H^{>}}{\partial^{>}p^{>}} = \frac{p^{>}}{m}, \quad \frac{\mathrm{d}p^{>}}{\mathrm{d}t} = -kv,$$

where one should note that the derivative can be assumed to be conventional, since the system is represented by the mutation of the Lie structure.

To achieve a representation of system (5.1) suitable for operator image, we need the following *classical, finite, Lie-admissible transformation genogroup*:

$$\text{(5.8)}\quad A(t) = \left(e^{-t\frac{\partial H}{\partial a^{\mu}}S^{>\mu\nu}\frac{\partial}{\partial a^{\nu}}}\right) A(0) \left(e^{\frac{\partial}{\partial a^{\nu}}{}^{<}S^{\nu\mu}\frac{\partial H}{\partial a^{\mu}}t}\right),$$

defined in the 12-dimensional *bimodular genophasespace* ${}^{<}T^{*}M \times T^{*}M^{>}$, with *infinitesimal Lie-admissible time evolution*

$$\begin{aligned} \text{(5.9)}\quad \frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{\partial A}{\partial a^{\mu}}({}^{<}S^{\mu\nu} - S^{>\mu\nu})\frac{\partial H}{\partial a^{\nu}} = \\ &= \left(\frac{\partial A}{\partial r^{k}}\frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial r^{k}}\frac{\partial A}{\partial p_k}\right) - \left(\frac{kv}{(\partial H/\partial p)}\right)\frac{\partial H}{\partial p}\frac{\partial A}{\partial p} = \\ &= [A, H] - kv\frac{\partial A}{\partial p}, \end{aligned}$$

where we have dropped the forward arrow for notational convenience, and $\omega^{\mu\nu}$ is the canonical Lie tensor, thus proving the Lie-admissibility of the S-tensors. In fact, the attached antisymmetric brackets $[A, H]$ are the conventional Poisson brackets, while $\{A, H\}$ are indeed symmetric brackets (as requested by Lie-admissibility), but they do not characterize a Jordan algebra (subsect. **1**˙3).

It is easy to see that the time evolution of the Hamiltonian is given by

$$\text{(5.10)}\quad \frac{\mathrm{d}H}{\mathrm{d}t} = -kv\frac{\partial H}{\partial p} = -kv^{2},$$

thus correctly reproducing behavior (5.2).

The operator image of the above dissipative system is straightforward. Physically, we are also referring to a first approximation of a massive and stable elementary particle, such as an electron, penetrating within hadronic matter (such as a nucleus). Being stable, the particle is not expected to "disappear" at the initiation of the dissipative force and be converted into "virtual states" due to the inability of represent such a force, but more realistically the particle is expected to experience a rapid dissipation of its kinetic energy and perhaps after that participate in conventional processes.

Alternatively, we can say that an electron orbiting in an atomic structure does indeed evolve in time with conserved energy, and the system is indeed Hamiltonian. But the idea that the same electron when in the core of a star also evolves with conserved energy is repugnant to reason. Rather than adapting nature to manifestly limited Hamiltonian theories, we seek their covering for the treatment of systems for which said theories were not intended for.

The problem is to identify forward and backward genounits and related genotopic elements $I^> = 1/T^>, ^< I = 1/^< T$ for which the following *operator Lie-admissible genogroup* now defined on a genomodule $^<\mathcal{H} \times \mathcal{H}^>$,

$$A(t) = (e^{iHT^> t})A(0)(e^{-it ^< TH}), \tag{5.11}$$

and the related infinitesimal form,

$$i\frac{\mathrm{d}A}{\mathrm{d}t} = A < H - H > A = A ^< TH - HT^> A, \tag{5.12}$$

correctly represent the dissipative system here considered.

By noting that the Lie brackets in eqs. (5.9) are conventional, we seek a realization of the genotopic elements for which the Lie brackets attached to the Lie-admissible brackets (5.12) are conventional and the symmetric brackets are Jordan-isotopic. A solution is then given by

$$T^> = 1 - \Gamma, \quad ^< T = 1 + \Gamma, \tag{5.13}$$

for which eq. (5.12) becomes

$$\begin{aligned} i\frac{\mathrm{d}A}{\mathrm{d}t} &= (AH - HA) - (A\Gamma H + H\Gamma A) = \\ &= [A, H] - \{A \hat{,} H\}, \end{aligned} \tag{5.14}$$

where $[A, H]$ are conventional Lie brackets as desired, and $\{A \hat{,} H\}$ are Jordan-isotopic brackets. The desired representation then occurs for

$$I^> = e^{(k/m)H^{-1}} = 1/T^>, \quad ^< I = e^{-H^{-1}(k/m)} = 1/^< T, \tag{5.15a}$$

$$i\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{kp^2}{m^2} = -kv^2. \tag{5.15b}$$

Note that the achievement of the above operator form of system (5.1) without the Lie-admissible structure would have been impossible, to our knowledge.

Despite its elementary character, the above illustration has deep implications. In fact, the above example constitutes the only known operator formulation of a dissipative system in which the *nonconserved* energy is represented by a *Hermitean* operator H, thus being an *observable* despite its nonconservative character. In all other cases existing in the literature the Hamiltonian is generally *non-Hermitean,* thus *non-observable.*

The latter occurrence may illustrate the reason for the absence of a consistent operator formulation of nonconservative systems throughout the 20-th century until the advent of Lie-admissible formulations.

5˙2. *Direct universality of Lie-admissible representations for nonconservative systems.* – We now show that the Lie-admissible formulations are "directly universal", namely, they provide a classical and operator representation of all infinitely possible (well-behaved) nonconservative systems of N particles (universality)

$$m_n \frac{\mathrm{d}v_{nk}}{\mathrm{d}t} + \frac{\partial V}{\partial r^k_n} = F^{\mathrm{NSA}}_{nk}(t, r, p, \dot{p}, ...), \quad n = 1, 2, 3, ..., N, \quad k = 1, 2, 3, \tag{5.16}$$

directly in the frame of the observer, *i.e.* without transformations of the coordinates of the experimenter to mathematical frames (direct universality).

An illustration is given by a massive object moving at high speed within a resistive medium, such as a missile moving in our atmosphere. In this case the resistive force is approximated by a power series expansion in the velocity truncated up to the 10-th power for the high speeds of contemporary missiles

$$m \frac{\mathrm{d}v}{\mathrm{d}t} = \Sigma_{\alpha=1,2,...,10} k_\alpha v^\alpha, \tag{5.17}$$

for which any dream of conventional Hamiltonian representation is beyond the boundary of science.

The direct universality of the Hamilton-Santilli genomechanics was proved in subsect. **3**˙3. The representation in geno-phase-space is characterized by the conventional Hamiltonian representing the physical total energy, and the genounit for forward motion in time representing the NSA forces, according to the equations

$$H = \Sigma_{n,k} \frac{p^2_{nk}}{2m_n} + V(r), \quad I^> = \begin{pmatrix} 1 & (\frac{F^{\mathrm{NSA}}}{(\partial H/\partial p)}) \\ 1 & 0 \end{pmatrix} \tag{5.18}$$

under which we have the equations of motion (for $\mu, \nu = 1, 2, 3, ...6N$)

$$\begin{aligned} \frac{\mathrm{d}a^{>\mu}}{\mathrm{d}t} &= \begin{pmatrix} \mathrm{d}r^{>k}_n/\mathrm{d}t \\ \mathrm{d}p^>_{nk}/\mathrm{d}t \end{pmatrix} = S^{>\mu\nu} \frac{\partial^> H^>}{\partial^> a^{>\nu}} = \\ &= \begin{pmatrix} 0 & -1 \\ 1 & (\frac{F^{\mathrm{NSA}}}{(\partial H/\partial p)}) \end{pmatrix} \begin{pmatrix} \partial^> H^>/\partial^> r^{>k}_n \\ \partial^> H^>/\partial^> p^>_{nk} \end{pmatrix}, \end{aligned} \tag{5.19}$$

the classical, finite, Lie-admissible genosgenogroup

$$A(t) = e^{-t \frac{\partial H}{\partial a^\mu} S^{>\mu\nu} \frac{\partial}{\partial a^\nu}} A(0) e^{\frac{\partial}{\partial a^\nu} {}^< S^{\nu\mu} \frac{\partial H}{\partial a^\mu} t}, \tag{5.20}$$

with infinitesimal time evolution

$$\begin{aligned} \frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{\partial A}{\partial a^\mu} ({}^< S^{\mu\nu} - S^{>\mu\nu}) \frac{\partial H}{\partial a^\nu} = \\ &= \left(\frac{\partial A}{\partial r^k_n} \frac{\partial H}{\partial p_{nk}} - \frac{\partial H}{\partial r^k_n} \frac{\partial A}{\partial p_{nk}} \right) - \left(\frac{km}{(\partial H/\partial p)} \right)^{nk} \frac{\partial A}{\partial p_{nk}} \frac{\partial H}{\partial p_{nk}} = \\ &= [A, H] + \{A, H\}, \end{aligned} \tag{5.21}$$

yielding the correct *nonconservation of the energy*

$$\frac{\mathrm{d}H}{\mathrm{d}t} = v^k F_k^{\mathrm{NSA}}. \tag{5.22}$$

The operator image can be characterized by the genounits and related genotopic elements

$$I^{>} = e^{\Gamma} = 1/T^{>}, \quad {}^{<}I = e^{-\Gamma} = 1/{}^{<}T, \quad \Gamma = H^{-1}(v_n^k F_{nk}^{\mathrm{NSA}})H^{-1}, \tag{5.23}$$

with finite Lie-admissible time evolution

$$A(t) = \exp[iHe^{-\Gamma}t]A(0)\exp[-ite^{+\Gamma}H] \tag{5.24}$$

and related Heisenberg-Santilli genoequations

$$\begin{aligned} i\frac{\mathrm{d}A}{\mathrm{d}t} &= A < H - H > A = [A, H] + \{A\hat{,}H\} = \\ &= (AH - HA) + (A\Gamma H + H\Gamma A), \end{aligned} \tag{5.25}$$

that correctly represent the time rate of variation of the nonconserved energy,

$$i\frac{\mathrm{d}H}{\mathrm{d}t} = v_n^k F_{nk}^{\mathrm{NSA}}. \tag{5.26}$$

The noninitiated reader should be aware that generally different genounits may be requested for different generators, as identified since refs. [11,18b].

In the latter operator case we are referring to an extended, massive and stable particle, such as a proton, penetrating at high energy within a nucleus, in which case the rapid decay of the kinetic energy is caused by contact, resistive, integrodifferential forces of nonlocal type, *e.g.*, because occurring over the volume of the particle.

The advantages of the Lie-admissible formulations over pre-existing representations of nonconservative systems should be pointed out. Again, a primary advantage of the Lie-admissible treatment is the characterization of the *nonconserved* Hamiltonian with a *Hermitean,* thus *observable* quantity, a feature generally absent in other treatments.

Moreover, the "direct universality" of Lie-admissible representations requires the following comments. Recall that coordinate transformations have indeed been used in the representation of nonconservative systems because, under sufficient continuity and regularity, the Lie-Koening theorem assures the existence of coordinate transformations $(r, p) \rightarrow (r'(r, p), p'(r, p))$ under which a system that is non-Hamiltonian in the original coordinates becomes Hamiltonian in the new coordinates (see ref. [6a] for details). However, the needed transformations are necessarily nonlinear with serious physical consequences, such as:

1) Quantities with direct physical meaning in the coordinates of the experimenter, such as the Hamiltonian $H(r, p) = \frac{p^2}{2m} + V(r)$, are transformed into quantities that, in the new coordinates, have a purely mathematical meaning, such as $H'(r', p') = N\exp[Mr'^2/p'^3], N, M \in R$, thus preventing any physically meaningful operator treatment.

2) There is the loss of meaningful experimental verifications, since it is impossible to place any measurement apparatus in mathematical coordinates such as $r' = K \log(Lr^3)$, $p' = P \exp[Qrp], L, P, Q \in R$.

3) There is the loss of Galileo's and Einstein's special relativity, trivially, because the new coordinates (r', p') characterize a highly *noninertial* image of the original inertial system of the experimenter.

All the above, and other insufficiencies are resolved by the Lie-admissible treatment of nonconservative systems.

5˙3. *Genotopies of Pauli matrices.* – Following the study of the nonconservation of the energy, the next important topic is to study the behavior of the conventional quantum spin under contact nonconservative forces. For this objective, it is most convenient to use the method of subsects. **4**˙2 and **4**˙3, namely, subject to the conventional Pauli's matrices to two different nonunitary transforms. To avoid un-necessary complexity, we select the following two simple matrices:

$$A = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}, \quad AA^\dagger \neq I, \;] \, BB^\dagger \neq I, \tag{5.27}$$

where a and b are non-null real numbers, under which we have the following forward and backward genounits and related genotopic elements:

$$I^> = AB^\dagger = \begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix}, \quad T^> = \frac{1}{(1-ab)} \begin{pmatrix} 1 & -b \\ -a & 1 \end{pmatrix}, \tag{5.28a}$$

$$^<I = BA^\dagger = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad ^<T = \frac{1}{(1-ab} \begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix}. \tag{5.28b}$$

The *forward and backward Pauli-Santilli genomatrices* are then given, respectively, by

$$\sigma_1^> = A\sigma_1 B^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \quad \sigma_2^> = A\sigma_2 B^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \tag{5.29a}$$

$$\sigma_3^> = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & b \\ a & -1 \end{pmatrix}, \qquad ^<\sigma_1 = B\sigma_1 A^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \tag{5.29b}$$

$$^<\sigma_2 = B\sigma_2 A^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \quad ^<\sigma_3 = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & a \\ b & -1 \end{pmatrix}, \tag{5.29c}$$

in which the direction of time is embedded in the structure of the matrices.

It is an instructive exercise for the interested reader to verify that *conventional commutation rules and eigenvalues of Pauli's matrices are preserved under forward and backward genotopies*,

$$\sigma_i^> > \sigma_j^> - \sigma_j^> > \sigma_i^> = 2i\epsilon_{ijk}\sigma_k^> \tag{5.30a}$$

$$\sigma_3^> > | >= \pm 1| >, \quad \sigma^{>^{2>}} > | >= 2(2+1)| >, \tag{5.30b}$$

$$^<\sigma_i >^< \sigma_j -^< \sigma_j >^< \sigma_i = 2i\epsilon_{ijk}^< \sigma_k \tag{5.30c}$$

$$< | <^< \sigma_3 =< | \pm 1, \; ; < | <^< \sigma^{2>} =< |(2(2+1). \tag{5.30d}$$

We can, therefore, conclude by stating that *Pauli's matrices can indeed be lifted in such an irreversible form to represent the direction of time in their very structure.*

Note, however, that expressions (5.30) are *Lie-isotopic* and not Lie-admissible and that conventional structure constants are fully preserved under isotopy [18b,18c]. Consequently, the conventional quantum spin is indeed conserved in expressions (5.30).

In order to study the behavior of spin in *nonconservative* conditions, it is necessary to lift the $SU(2)$-spin *group* into a Lie-admissible genogroup, resulting in Lie-admissible (rather than Lie-isotopic), infinitesimal form by following the rules of the Lie-admissible theory presented and illustrated in preceding parts of this paper.

The above procedure results in Lie-admissible brackets of the type $\sigma_i < \sigma_j - \sigma_j > \sigma_i = C_{ijk}\sigma_k$, for which there is a realistic possibility of preserving the conventional structure constants in the underlying geno-bi-representation space, *i.e.* $C_{ijk} = 2i\epsilon_{ijk}$. This is due to the fact that the backward (forward) genoproduct $\sigma_i < \sigma_j$ ($\sigma_j > \sigma_i$) is computed with respect to the backward (forward) genounit ${}^{<}I$ ($I^{>}$), rather than the conventional unit I, as a result of which the value of the backward genoproduct can be equal to minus the value of the forward one, as typical of all Lie bimodules, *i.e.* $\sigma_i < \sigma_j|_{^{<}I} = -\sigma_i > \sigma_j|_{I^{>}}$.

However, when the above Lie-admissible geno-bi-structure is projected in our space (that is, both forward and backward products are computed with respect to the trivial unit I), the Lie bimodule character is generally lost and the eigenvalues of the spin are predicted to become *continuously varying in time*, as expected for the spin of an electron in the hyperdense medium in the core of a star or a black hole.

The detailed study of the above Lie-admissible nonconservative lifting of the quantum spin is left as an intriguing open problem for young minds of any age.

5·4. *Genotopies of the Minkowski space-time.* – One of the fundamental axiomatic principles of hadronic mechanics is that irreversibility can be directly represented with the background geometry and, more specifically, with the metric of the selected geometry. This requires the necessary transition from the conventional *symmetric* metrics used in the 20-th century to covering *nonsymmetric* genometric.

To show this structure, we study in this section the genotopy of the conventional Minkowskian space-time and related geometry with the conventional metric $\eta = \text{Diag}\,(1,1,1,-1)$ and related space-time elements $x^2 = x^\mu \eta_{\mu\nu} x^\nu, x = (x^1, x^2, x^3, x^4), x^4 = ct, c = 1$. For this purpose, we introduce the following four-dimensional non-Hermitean, nonsingular and real-valued forward and backward genounits:

$$I^{>} = CD^\dagger = 1/T^{>}, \quad {}^{<}I = DC^\dagger = 1/{}^{<}T, \quad CC^\dagger \neq I, \quad DD^\dagger \neq I, \tag{5.31}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{5.32}$$

where $p \neq q$ are non-null real numbers, under which we have the following forward and backward genotopy of the Minkowskian line element:

$$\begin{aligned} x^2 \to x^{>^{2>}} &= Cx^2D^\dagger = C(x^t\eta x)D^\dagger = \\ &= (C^t x^t D^{t\dagger})(CD^\dagger)^{-1}(C\eta D^\dagger)(CD^\dagger)^{-1}(CxD^\dagger) = \\ &= (x^t I^{>})T^{>}\eta^{>}T^{>}(I^{>}x) = x^\mu \eta^{>}_{\mu\nu} x^\nu = \end{aligned} \tag{5.33a}$$

$$= (x^1x^1 + x^1qx^3 + x^2x^2 + x^3x^3 + x^1px^4 - x^4x^4),$$

(5.33b)
$$Dx^2C^\dagger = D(x^t\eta x)C^\dagger =$$
$$= (x^{t^<}I)^<T^<\eta^<T(^<Ix) = x^{\mu^<}\eta_{\mu\nu}x^\nu =$$
$$= (x^1x^1 + x^1px^3 + x^2x^2 + x^3x^3 + x^1qx^4 - x^4x^4),$$

resulting in the forward and backward nonsymmetric genometrics

(5.34)
$$\eta^> = \begin{pmatrix} 1 & 0 & q & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ q & 0 & 0 & 1 \end{pmatrix}, \quad {}^<\eta = \begin{pmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

exactly as desired.

Note that irreversibility selects a mutation of the line elements along a pre-selected direction of space and time.

Note also that the quantities p and q can be functions on the local space-time variables, in which case the resulting *Minkowskian genogeometry* can be equipped by a suitable lifting of the machinery of the Riemannian geometry (see ref. [16] for the isotopic case).

It should be indicated that the above irreversible formulation of space-time has intriguing implications for the mathematical model known as *geometric locomotion* studied in detail in monograph [22] (see also studies [73] via the isotopies of the Minkowskian geometry. In fact, a main unresolved problem is the directional deformation of the geometry as needed to permit the geometric locomotion in one preferred direction of space. An inspection of the mutated line elements (5.33) clearly shows that the genotopies are preferable over the isotopies for the geometric locomotion, as well as, more generally, for a more realistic geometric characterization of irreversible processes.

In a subsequent paper the author hopes to present the explicit form of the Lorentz-Santilli genotransformations leaving invariant genoelements (5.33) and related genotopy of special relativity, again, for the purpose of showing that irreversibility can indeed be embedded in the basic axioms of physical theories (see ref. [18] for isospecial relativity).

5˙5. *Genotopies of Dirac's equation.* – To complete the illustrations in particle physics, we now outline the simplest possible genotopy of Dirac's equation via the genotopies of the preceding two sections, one for the spin content of Dirac's equation and the other for its space-time structure. Also, we shall use Dirac's equation in its isodual re-interpretation representing a direct product of one electron and one positron, the latter without any need of second quantization (see monograph [22] for detail). In turn, the latter re-interpretation requires the use of the *isodual transform* $A \rightarrow A^d = -A^\dagger$ as being distinct from Hermitean conjugation. Under the above clarifications, the *forward Dirac genoequation* here referred to can be written as

(5.35a)
$$(\eta^{>\mu\nu}\gamma^>_\mu T^> p^>_\nu - im)\, T^>|\psi^> >= 0$$

(5.35b)
$$p^>_\nu T^>|\psi^> >= -i\frac{\partial^>}{\partial^> x^{>\nu}}|\psi^> >= -iI^>\frac{\partial}{\partial x^>}|\psi^> >,$$

with *forward genogamma matrices*

(5.36a)
$$\gamma^>_4 = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix}\begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}\begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \begin{pmatrix} AA^d & 0 \\ 0 & -B^dB \end{pmatrix},$$

$$(5.36b)\ \gamma_k^> = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} \begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & A\sigma_k B^\dagger \\ B\sigma_k^d A^d & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_k^> \\ {}^<\sigma_k^d & 0 \end{pmatrix},$$

$$(5.36c)\ \{\gamma_\mu^> \hat{,} \gamma_\nu^>\} = \gamma_\mu^> T^> \gamma_\nu^> + \gamma_\nu^> T^> \gamma_\mu^> = 2\eta_{\mu\nu}^>,$$

where $\eta_{\mu\nu}^>$ is given by the same genotopy of eqs. $(5.36a)$.

Interested readers can then construct the backward genoequation. They will discover in this way a new fundamental symmetry of Dirac's equation that remained undiscovered throughout the 20-th century, its *iso-self-duality* (invariance under isoduality) that is now playing an increasing role for realistic cosmologies, those inclusive of antimatter, or serious unified theories that must also include antimatter to avoid catastrophic inconsistencies [22].

Note that, while the electron is moving forward in time, the positron is moving backward although referred to a negative unit of time, as a necessary condition to avoid the inconsistencies for negative energies that requested the conjecture of the "hole theory" (see monograph [22] for brevity).

5˙6. *Applications to thermodynamics*. – An additional scientific imbalance of the 20-th century has been the lack of interconnections between thermodynamics and classical and quantum Hamiltonian mechanics. This is due to the fact that the very notion of entrophy, let alone all thermodynamical laws, are centrally dependent on irreversibility, while classical and quantum Hamiltonian mechanics are structurally reversible (since all known potentials are reversible in time).

As recalled in sect. **1**, this lack of interconnection was justified in the 20-th century via the belief that the nonconservative forces responsible for irreversibility, according to Lagrange and Hamilton, are "fictitious" in the sense that they only exist at the classical level and they "disappear" when passing to elementary particles, since the latter were believed to be completely reversible. In this way, thermodynamics itself was turned into a sort of "fictitious" discipline without intrinsic reality.

This imbalance has been resolved by Lie-admissible formulations. In fact, theorem 1.1 has established that, far from being "fictitious", nonconservative forces originate at the ultimate level of nature, that of elementary particles in conditions of mutual penetration causing contact nonpotential (NSA) interactions. The insufficiency rested in the inability by quantum mechanics to represent nonconservative forces, rather than in nature. In fact, hadronic mechanics was proposed and developed precisely to reach an operator representation of the nonconservative forces originating irreversibility along the legacy of Lagrange and Hamilton.

As a result of the efforts presented in this paper, we now possess not only classical and operator theories, but more particularly we have a *new mathematics*, genomathematics, whose basic axioms are not invariant under time reversal beginning from the basic units, numbers and differentials.

Consequently, it is now possible to study, apparently for the first time, the expected interplay between thermodynamics and classical as well as operator mechanics according to the research initiated by J. Dunning Davies [74]. For brevity, in this section we can only indicate the initial lines.

Let us use conventional thermodynamical symbols, the classical form of thermodynamics, and the simple construction of irreversible formulations via two different complex valued quantities A and B. Then, the first law of thermodynamics can be lifted from its conventional formulation, that via reversible mathematics, into the form permitted by genomathematics

$$Q \to Q^> = AQB^\dagger = QI^>, \quad U \to U^> = AUB^\dagger = UI^>, \quad \text{etc.}, \tag{5.37a}$$

$$dQ = dU + pdV \quad \to \quad d^>Q^> = d^>U^> + p^> > d^>V^>, \tag{5.37b}$$

where, in the absence of operator forms, Hermitean conjugation is complex conjugation. For the second law we have

$$dQ = TdS \quad \to \quad d^>Q^> = T^> > d^>S^>, \tag{5.38}$$

thus implying that

$$TdS = dU + pdV \quad \to \quad T^> > d^>S^> = d^>U^> + p^> > d^>V^>. \tag{5.39}$$

As one can see, genomathematics permits the *first known formulation of entrophy with a time arrow,* the only causal form being that forward in time. When the genounit does not depend on the local variables, the above genoformulation reduces to the conventional one identically, *e.g.*,

$$\begin{aligned} T^> > d^>S^> &= (TI^>)I^{>-1}[I^{>-1}d(SI^>) = TdS = \\ &= I^{>-1}d(VI^>) + (pI^>)I^{>-1}d(VI^>) = dU + pdV. \end{aligned} \tag{5.40}$$

This confirms that genomathematics is indeed compatible with thermodynamical laws.

However, new vistas in thermodynamics are permitted when the genounit is dependent on local variables, in which case reduction (5.40) is no longer possible. An important case occurs when the genounit is explicitly dependent on the entrophy. In this case the l.h.s. of eq. (5.40) becomes

$$TdS + TS(I^{>-1}dI^>) = dU + pdV. \tag{5.41}$$

We then have new thermodynamical models of the type

$$I^> = e^{f(S)}, \;\; T^> > d^>S^> = T(1 + S\frac{\partial f(S)}{\partial S})dS = dU + pdV, \tag{5.42}$$

permitting thermodynamical formulations of the behavior of anomalous gases (such as magnegases [21]) via a suitable selection of the $f(S)$ function and its fit to experimental data. Needless to say, equivalent models can be constructed for an explicit dependence of the genounit from the other variables. For these and other aspects we have to refer the interested reader to ref. [74].

5·7. *Ongoing applications to new clean energies.* – The societal, let alone scientific implications of the proper treatment of irreversibility are rather serious. Our planet is afflicted by increasingly catastrophic climactic events mandating the search for basically new, environmentally acceptable energies.

All known energy sources, from the combustion of carbon dating to prehistoric times to the nuclear energy, are based on irreversible processes. By comparison, all established doctrines of the 20-th century, such as quantum mechanics and special relativity, are reversible in time.

It is then easy to see that *the serious search for basically new energies requires basically new theories that are as structurally irreversible as the process they are expected to describe.* At any rate, all possible energies and fuels that could be predicted by quantum mechanics and special relativity were discovered by the middle of the 20-th century. Hence, the insistence in continuing to restrict new energies to verify preferred reversible doctrines may cause a condemnation by posterity due to the environmental implications.

An effective way to illustrate the need for new irreversible theories is given by nuclear fusions. All efforts to date in the field, whether for the "cold fusion" or the "hot fusion", have been mainly restricted to verify quantum mechanics and special relativity. However, *whether "hot" or "cold", all nuclear fusion processes are irreversible, while quantum mechanics and special relativity are reversible.*

It has been shown in ref. [75] that the failure to date by both the "cold" and the "hot" fusions to achieve industrial value is due precisely to the treatment of irreversible nuclear fusions with reversible mathematical and physical methods.

In the event of residual doubt due to protracted use of preferred theories, it is sufficient to compute the quantum-mechanical probability for two nuclei to "fuse" into a third one, and then compute its time reversal image. In this way the serious scholar will see that special relativity and quantum mechanics may predict the *spontaneous disintegration of a nucleus into the original nuclei*, namely, a prediction outside the boundary of science due to the evident violation of causality.

The inclusion of irreversibility in quantitative studies of new energies suggests the development, already partially achieved at the industrial level (see Chapt. 8 of ref. [19]), of the new, controlled "intermediate fusion" of light nuclei [75], that is, a fusion occurring at minimal threshold energies needed to: 1) verify conservation laws; 2) expose nuclei as a pre-requisite for their fusion (a feature absent in the "cold fusion" due to insufficient energies), and 3) prevent uncontrollable instabilities (as occurring at the very high energies of the "hot fusion").

It is hoped that serious scholars will participate with independent studies on the irreversible treatment of new energies, as well as on numerous other open problems in Lie-admissible formulations, because, in the final analysis, lack of participation in basic advances is a gift of scientific priorities to others.

* * *

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